MATH538 - ALGEBRAIC TOPOLOGY II - SPRING 2018
HOMEWORK 4

Due: May 21, 2018

Q1. Show that if M is a compact contractible $n-$manifold for $n \geq 1$, then ∂M is a homology $(n-1)-$sphere. That is, show that

$$H_i(\partial M; \mathbb{Z}) \cong H_i(S^{n-1}; \mathbb{Z}) \quad \forall i.$$

Q2. Can there be connected CW-complexes X and Y such that

$$H_i(X; \mathbb{Z}) = H_i(Y; \mathbb{Z}), \quad \forall i$$

but where the isomorphisms can not be induced by any continuous function from $X \to Y$ or from $Y \to X$? If yes, construct such two complexes and verify the claim.

Q3. Suppose that X is a path-connected manifold of dimension $n > 1$ such that its fundamental group has order 537. Prove that X is orientable.

Q4. Let M be an n-dimensional compact manifold such that

$$H_i(M; \mathbb{Z}_2) = H_i(S^n; \mathbb{Z}_2), \quad \forall i.$$

Prove that M is orientable.