## MATH538 - ALGEBRAIC TOPOLOGY II - SPRING 2018 HOMEWORK 3

## Due: April 25, 2018

**Q1.** Compute the cohomology ring structure of  $H^*(T^n; Z)$  where

$$T^n = S^1 \times S^1 \cdots \times S^1$$

(n factors) is the *n*-torus.

**Q2.** Show that if a topological space is a union of two contractible sets, then for any two cohomology classes of positive degrees, their cup product vanishes.

**Q3.** Let X be any topological space, and let  $t_0 \in S^1$  be a point on the unit circle. Prove, for each i, that the connecting homomorphism

$$\delta: H^{i}(X \times \{t_{0}\}; R) \longrightarrow H^{i+1}(X \times S^{1}, X \times \{t_{0}\}; R)$$

in the long exact sequence of the pair  $(X \times S^1, X \times \{t_0\})$  is trivial for any coefficient ring R.

**Q4.** Show that for any map  $f: S^4 \to S^2 \times S^2$ , the induced map in the integral homology  $f_*: H_4(S^4) \to H_4(S^2 \times S^2)$  is trivial.