MATH538 - ALGEBRAIC TOPOLOGY II - SPRING 2018
HOMEWORK 2

Due: April 04, 2018

Q1. Let X be the space obtained by attaching two discs to S^1, where the first disk D_1 is attached via the map $\partial D_1 = S^1 \to S^1$ given by $z \to z^4$ and the second disk D_2 is attached via the map $\partial D_2 = S^1 \to S^1$ given by $z \to z^7$. (Here we consider $S^1 \subset \mathbb{C}$ and z is the coordinate on $\mathbb{C}$.)
(**a**) Compute the integral homology groups of X.
(**b**) Compute the cohomology groups of X with $\mathbb{Z}_2$-coefficients.
(**c**) Compute the relative cohomology groups $H^*(X, S^1; \mathbb{Z}_2)$.
(**d**) Is X homeomorphic to S^2? Explain?

Q2. Let $p > 1, q$ be relatively prime positive integers, and let $L(p,q)$ denote the *Lens space* obtained by gluing the boundaries of two solid tori

$$T_1 \approx S^1 \times D^2, \;\; T_2 \approx S^1 \times D^2$$

together such that the meridian curve of T_1 (i.e., the circle $\{pt\} \times \partial D^2$ on ∂T_1) goes to a (p,q)-curve on ∂T_2, where a (p,q) -curve wraps around the longitude curve (i.e., the circle $S^1 \times \{pt\}$ on ∂T_2) p times and around the meridian on ∂T_2 q times.
a) Compute the integral homology groups $H_*(L(p,q))$.
b) Compute $H^*(L(p,q)\;; \mathbb{Z}_p)$.
c) Compute $H^*(L(p,q)\;; \mathbb{Q})$.

Q3. Compute the cohomology ring of the two-torus $\mathbb{T}^2$.

Q4. Let L be the union of two once linked circles in S^3, and also let L' be the union of two unlinked circles in S^3. Show that the cohomology groups of $S^3 - L$ and $S^3 - L'$ are isomorphic, but the cohomology rings are not.