MATH538 - ALGEBRAIC TOPOLOGY II - SPRING 2018
HOMEWORK 1

Due: March 12, 2018

Q1. Compute the cohomology groups of the 2-torus $\mathbb{T}^2$, the projective plane $\mathbb{R}P^2$ and the Klein Bottle $\mathbb{K}B^2$ with $\mathbb{Z}$ and $\mathbb{Z}_2$ coefficients using

a) directly the definitions of cohomology groups,

b) their integral homology groups and the universal coefficient theorem.

Q2. Let $X = M(m,n)$ be the (so called) Moore space obtained from S^n by attaching a cell e^{n+1} by a map of degree m. Show that the inclusion map

$$i : S^n \to X$$

induces the trivial map on reduced cohomology but not on $H_n(\_\,;\mathbb{Z})$.

Q3. Show that if $f : S^n \to S^n$ has degree d, then the induced map

$$f^* : H^n(S^n;G) \to H^n(S^n;G)$$

is given by multiplication by d for any abelian group G.

Q4. **a**) Compute the cohomology groups $H^*(\mathbb{R}P^n\;;G)$ of the projective space $\mathbb{R}P^n$ for $G = \mathbb{Z}$ and $G = \mathbb{Z}_2$

b) Show that there is no map $\mathbb{R}P^n \to \mathbb{R}P^m$ inducing a nontrivial map $H^1(\mathbb{R}P^m, \mathbb{Z}_2) \to H^1(\mathbb{R}P^n, \mathbb{Z}_2)$, provided that $n > m$.