Q1. [Prof. Liberzon] Consider the second-order system

$$\dot{x} = \begin{bmatrix} -2 & 2 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u\,, \qquad |u| \leq 1.$$

We wish to drive the state $x = [x_1\ x_2]^T$ from an arbitrary initial condition $x(0)$ to the origin in minimum time. Show that the optimal control is of bang-bang type and switches at most once. Obtain an expression (of the form $f(x_1, x_2) = 0$) for the switching curve.

Q2. For the scalar system $\dot{x} = -x + u - 1$ and the initial condition $x(0) = 1$ the below cost is to be minimized.

$$J = x(1)^2 + \int_0^1 u(t)^2 dt\,.$$

Find the optimal control $u^*(t)$ and the optimal trajectory $x^*(t)$.

Q3. Problem 5-11.

Q4. Problem 5-17.