EE554 Second Midterm Exam 2 May 2019

First name:______________________________

Last name:______________________________

Student ID:______________________________

Signature:______________________________

Read before you start:

- There are four questions.
- The examination is closed-book.
- No computer/calculator is allowed.
- The duration of the examination is 100 minutes.
- Besides correctness, the CLARITY of your presentation will also be graded.

Q1	**Q2**	**Q3**	**Q4**	**Total**

Q1. 20%

The state of the scalar system $\dot{x} = x + u$ starting from the initial condition $x(0) = 30$ is to be driven to the origin in $\ln 8$ seconds, i.e., $x(\ln 8) = 0$, while minimizing the cost

$$J = \int_0^{\ln 8} \frac{1}{4} u^4 dt\,.$$

Find the resulting optimal trajectory $x^*(t)$.

Q2. 30%

The trajectory $(x_1^*(t),\, x_2^*(t))$ of the following system with bounded input $u \in [-1,\, 1]$

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + u \end{aligned}$$

evolves throughout the interval $t \in [0,\, \pi]$ under the control $u^*(t)$ that minimizes the cost

$$J = \frac{1}{2}\left[x_1(\pi)^2 + x_2(\pi)^2\right] .$$

It is measured that $x_1^*(\pi) = 0$ and $x_2^*(\pi) = -1$. Find the initial conditions $x_1^*(0)$ and $x_2^*(0)$.

Q3. 30%

Consider the linear system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

where the triple $(C,\ A,\ B)$ is both observable and controllable.

(a) Show that the triple $(C,\ [A + \alpha I],\ B)$ is both detectable and stabilizable for all $\alpha \in \mathbb{R}$.

(b) Let $\alpha > 0$ and R be symmetric positive definite. Show that the below cost

$$J = \int_0^\infty e^{2\alpha t}(y^T y + u^T R u)dt$$

admits a minimizer in the form $u_{\min}(t) = -Kx(t)$ where K is a constant feedback gain. *Hint: Employ the change of variables $z := e^{\alpha t}x$ and $v := e^{\alpha t}u$.*

(c) What can be said about the eigenvalue locations of the closed-loop system matrix $A - BK$?

Q4. 20%

Consider the linearization of the (normalized) pendulum system without friction

$$\ddot{y} + y = u$$

where y is the (angular) position of the pendulum and u is the torque applied to the system. Cost to be minimized is

$$J = \int_0^\infty (3y^2 + 3\dot{y}^2 + u^2)dt\,.$$

(a) Find J_{min} in terms of $y(0)$ and $\dot{y}(0)$.

(b) Let the optimal control law be $u_{\text{min}} = -k_1 y - k_2 \dot{y}$. Find k_1 and k_2.