

First name:_____

Last name:_____

Student ID:_____

Signature:_____

Read before you start:

- There are four questions.
- The examination is closed-book.
- No computer/calculator is allowed.
- The duration of the examination is 100 minutes.
- Besides correctness, the CLARITY of your presentation will also be graded.

Q1	Q2	Q3	Q4	Total

(a) Write the variation δJ of the functional

$$J(x_1, x_2) = \int_{t_0}^{t_f} (x_1^2 + x_1 x_2 + x_2^2 + 2\dot{x}_1 \dot{x}_2) dt$$

where the endpoints (initial time, initial state, final time, final state) are given.

(b) Find the extremal $x^*(t)$ of the following functional

$$J(x) = \int_0^7 (x^2 + 2x\dot{x} - \dot{x}^2) dt \quad \text{subject to} \quad x(0) = 1 \text{ and } x(7) = 0.$$

(c) Find the extremal $x^*(t)$ of the following functional

$$J(x) = \int_0^1 \left(\frac{1}{2} \dot{x}^2 + x\dot{x} + \dot{x} + x \right) dt \quad \text{subject to} \quad x(0) = \frac{1}{2} \text{ and } x(1) \text{ free.}$$

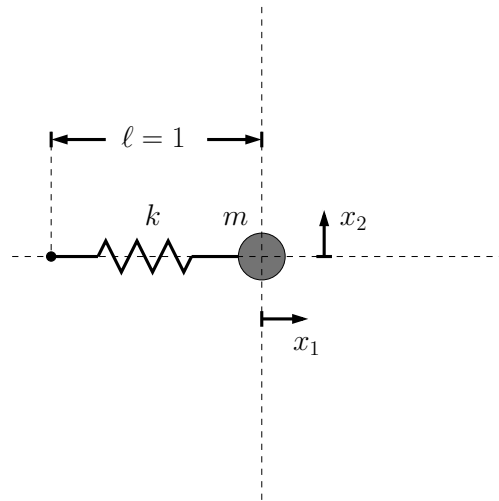
Q2.

25%

For the scalar system $\dot{x} = u$ starting from the initial condition $x(0) = 1$ the following cost

$$J(u) = x(1)^2 + \int_0^1 (x^2 + u^2) dt$$

is to be minimized. Find the optimal trajectory $x^*(t)$.



Consider the frictionless planar mass-spring system shown at rest in the figure. (Note that the equilibrium shown in the figure is not the only equilibrium.) The body with mass m is connected to the spring with spring constant k . The other end of the spring is fixed to the frame of reference as shown. This body is allowed to move on the plane, where the displacement from the shown equilibrium to east is denoted by x_1 and to north by x_2 . Obtain the dynamics of this system using the Euler-Lagrange equation.

Consider the scalar system $\dot{x} = ax + u$ and the associated optimal cost

$$J^*(t_0, x(t_0)) = \min_{u(\cdot)} \int_{t_0}^{\infty} (x(t)^2 + u(t)^2) dt.$$

It is known that $J^*(t, x) = px^2$ for some constant p .

- (a) Find p (in terms of a) using HJB equation.
- (b) Show that optimal trajectories satisfy $|x^*(t)| \leq |x(0)|e^{-t}$ for all a .