Vibration of a Mass-Spring-Damper System

$$m\cdot\frac{d^2}{dt^2}x + c\cdot\frac{d}{dt}x + k\cdot x = f(t) = \sin(5\cdot t)$$

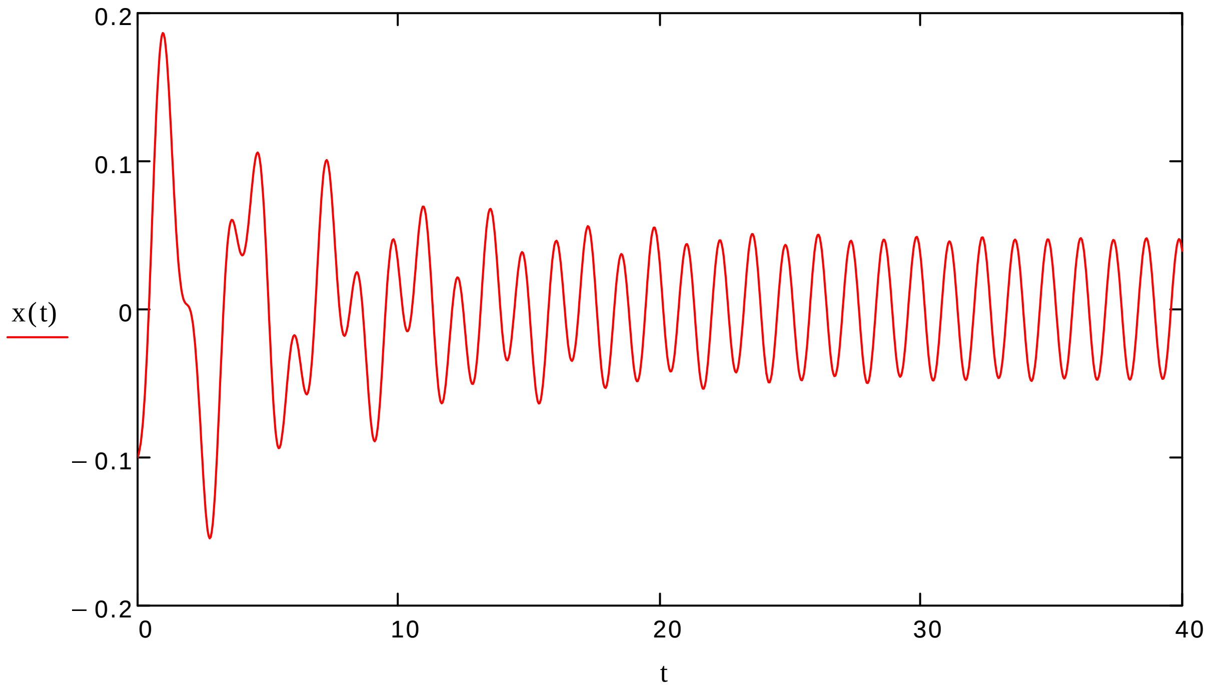
$t_f := 40$ $m := 1$ $c := 0.3$ $k := 4$ $\omega := 5$

Given

$$m\cdot\frac{d^2}{dt^2}x(t) + c\cdot\frac{d}{dt}x(t) + k\cdot x(t) = \sin(\omega\cdot t)$$

$x(0) = -0.1$ $x'(0) = 0.05$

$x := \text{Odesolve}(t, t_f)$



Other Solvers can as well be used such as

Adams, rkfixed, Rkadapt, Bulstoer, Radau

ODEs should be first order

Let $x = y_1$ $\frac{d}{dt}x(t) = y_2$

$$\frac{d}{dt}y_1(t) = y_2$$

$$\frac{d}{dt}y_2(t) = \frac{1}{m}\cdot\left(\sin(\omega\cdot t) - c\cdot y_2(t) - k\cdot y_1(t)\right)$$

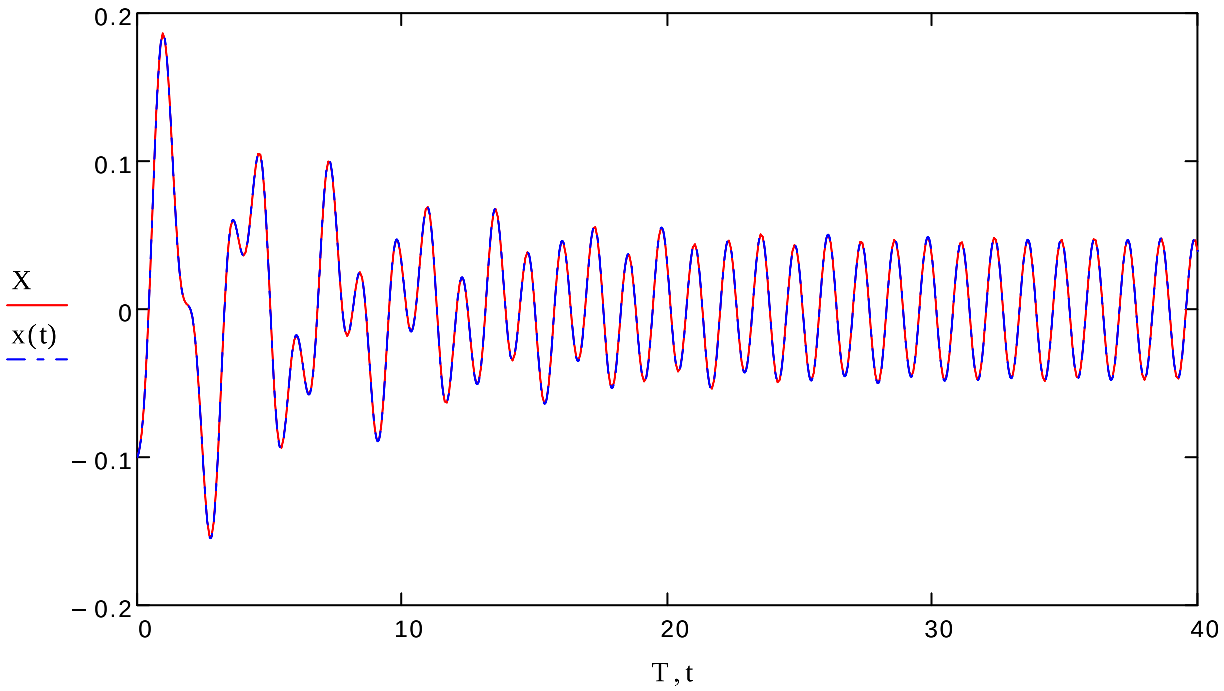
Initial Conditions $IC := \begin{pmatrix} -0.1 \\ 0.05 \end{pmatrix}$ $\begin{pmatrix} x(0) = y_1(0) \\ x'(0) = y_2(0) \end{pmatrix}$

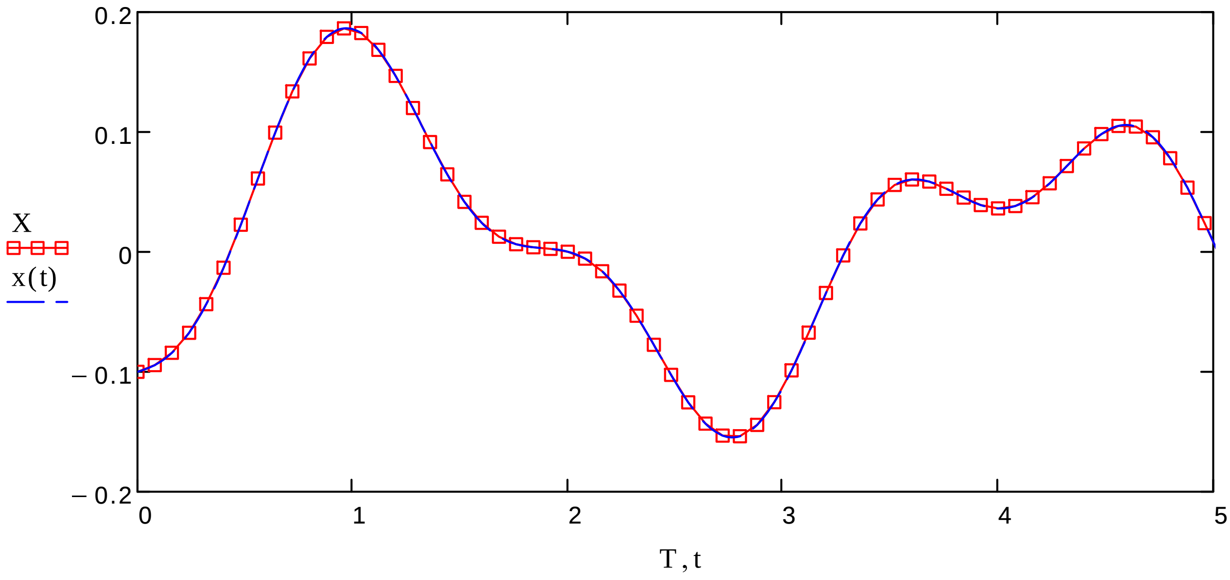
Set of ODEs $D(t,y) := \begin{bmatrix} y_1 \\ \frac{1}{m} \cdot (\sin(\omega \cdot t) - c \cdot y_1 - k \cdot y_0) \end{bmatrix}$

$t_s := 0$ $t_f = 40$ $NN := 500$ The integer number of discretization intervals (NN-1) used to interpolate the solution function.

$S := \text{Adams}(IC, t_s, t_f, NN, D)$

$X := S^{\langle 1 \rangle}$ $T := S^{\langle 0 \rangle}$ $X' := S^{\langle 2 \rangle}$ $t := t_s, t_s + 0.01 .. t_f$





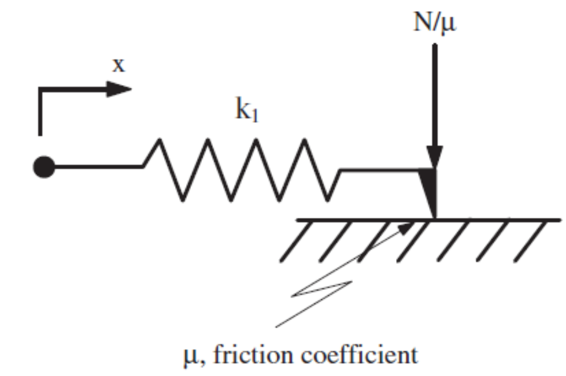
For Stiff Problems the Following Solvers can be Used

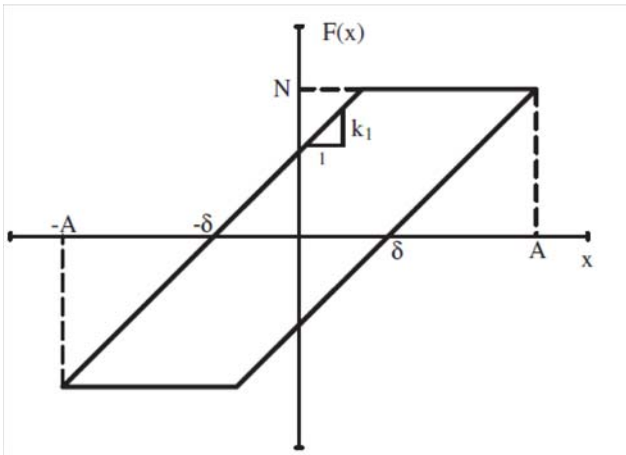
BDF, Radau, Stiffb, Stiffr

AdamsBDF : is a hybrid solver. It uses Adams for non-stiff problems and BDF for stiff problems. It automatically determines whether a system is stiff or non-stiff.

Friction element given below can be represented by $\frac{d}{dt}f(t) = k\cdot\left(\frac{d}{dt}u(t) - \frac{1}{\alpha}\cdot\operatorname{atanh}\left(\frac{f(t)}{\mu\cdot N}\right)\right)$

where f(t) is the friction force and u(t) is the input motion (x) to the friction element.





$NN := 1000$

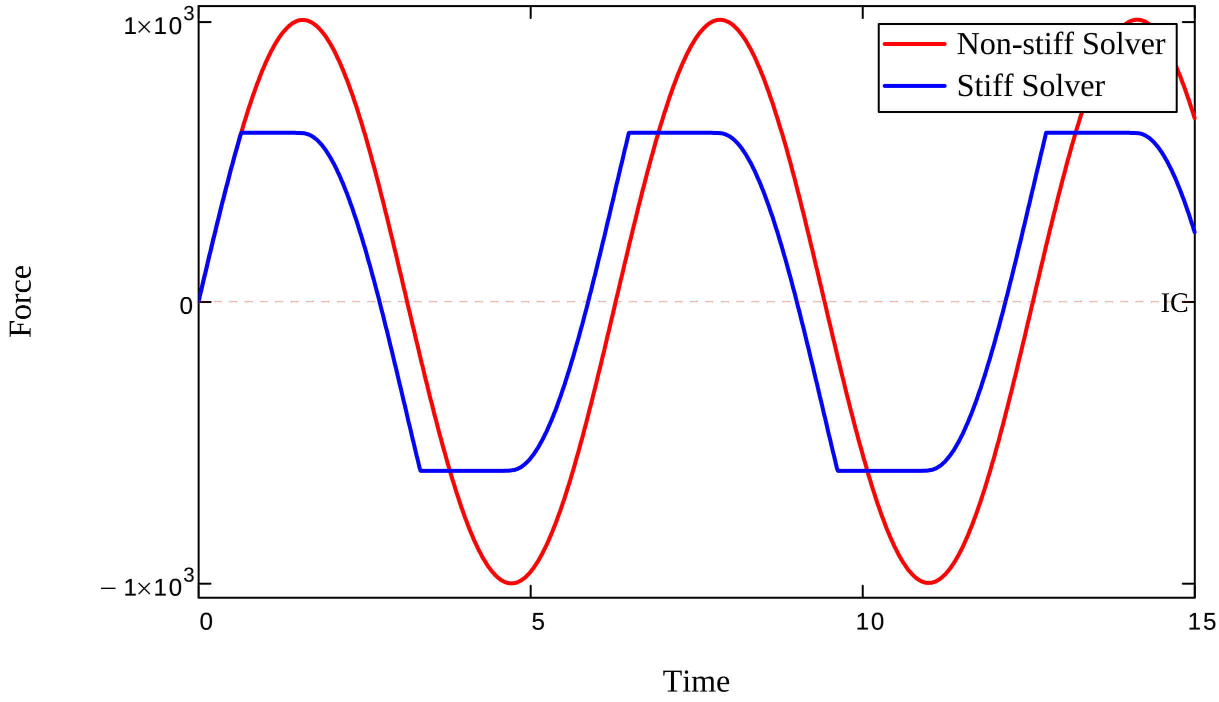
$k := 10^3$ $\mu := 1$ $N := 600$ $u(t) := 1\cdot\sin(t)$ $\alpha := 10^4$ $N_n := 10^3$

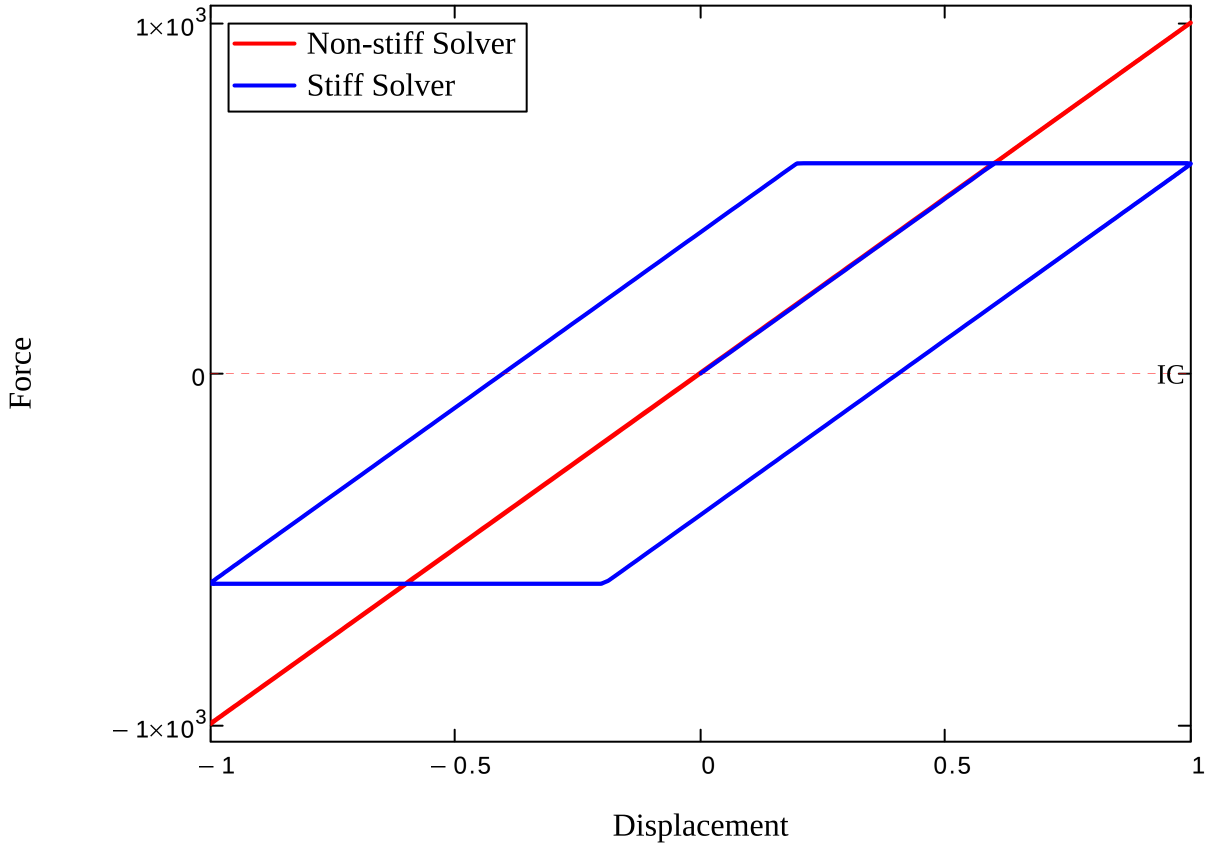
$F(t,f) := k\cdot\left(\frac{d}{dt}u(t) - \frac{1}{\alpha}\cdot\tan\left(\frac{\pi}{2}\cdot\frac{f}{\mu\cdot N}\right)\right)$ $t_0 := 0$ $f_0 := 0$ $t_1 := 15$

$D(t,f) := k\cdot\left(\frac{d}{dt}u(t) - \frac{1}{\alpha}\cdot\tan\left(\frac{\pi}{2}\cdot\frac{f}{\mu\cdot N}\right)\right)$ $IC := 0$ $t_s := 0$ $t_f := 15$

$S := \operatorname{rkfixed}(IC, t_s, t_f, NN, D)$ $S_2 := \operatorname{BDF}(IC, t_s, t_f, NN, D)$

$Y1 := S^{\langle 1\rangle}$ $T1 := S^{\langle 0\rangle}$ $Y2 := S_2^{\langle 1\rangle}$ $T2 := S_2^{\langle 0\rangle}$





As can be seen from the solutions, the non-stiff solver "rkfixed" cannot detect the stick slip transitions correctly. Therefore, one should use a stiff solver such as "BDF" in order to obtain the correct solution of a stiff problem. For stiff problems, non-stiff solvers require very small time steps which are not practical in most situations.