Consider a stress state in 2D

$$\sigma = \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix} \quad \Rightarrow \quad \sigma := \begin{pmatrix} -2.8 & 7.2 \\ 7.2 & 6.4 \end{pmatrix} \cdot MPa$$

Principal Stresses are the eigenvalues $\Rightarrow$ $\text{eigenvals}(\sigma) = \begin{pmatrix} -6.744 \\ 10.344 \end{pmatrix} MPa$

Consider a 5 dof mechanical system

$k_1 := 40000 \quad k_2 := 25000 \quad k_3 := 27000 \quad k_4 := 32000 \quad k_5 := 20000 \quad k_6 := 10000$

$m_1 := 20 \quad m_2 := 10 \quad m_3 := 25 \quad m_4 := 30 \quad m_5 := 20$

$$K := \begin{pmatrix} k_1 + k_2 & -k_2 & 0 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 & 0 \\ 0 & 0 & -k_4 & k_4 + k_5 & -k_5 \\ 0 & 0 & 0 & -k_5 & k_5 + k_6 \end{pmatrix} \quad \text{N/m}$$

$$M := \text{diag}\left[\begin{pmatrix} m_1 & m_2 & m_3 & m_4 & m_5 \end{pmatrix}^T\right] \quad \text{kg}$$

$$K = \begin{pmatrix} 6.5 \times 10^4 & -2.5 \times 10^4 & 0 & 0 & 0 \\ -2.5 \times 10^4 & 5.2 \times 10^4 & -2.7 \times 10^4 & 0 & 0 \\ 0 & -2.7 \times 10^4 & 5.9 \times 10^4 & -3.2 \times 10^4 & 0 \\ 0 & 0 & -3.2 \times 10^4 & 5.2 \times 10^4 & -2 \times 10^4 \\ 0 & 0 & 0 & -2 \times 10^4 & 3 \times 10^4 \end{pmatrix}$$

$$M = \begin{pmatrix} 20 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 25 & 0 & 0 \\ 0 & 0 & 0 & 30 & 0 \\ 0 & 0 & 0 & 0 & 20 \end{pmatrix}$$

to determine the natural frequencies and mode shapes of a system we need to determine eigenvalues and eigenvectors of $M^{-1} \cdot K$

$$\lambda := \mathrm{eigenvals}\left(M^{-1}\cdot K\right) \qquad \lambda = \begin{pmatrix} 6.786\times 10^{3} \\ 3.353\times 10^{3} \\ 2.321\times 10^{3} \\ 228.015 \\ 1.355\times 10^{3} \end{pmatrix}$$

$$\lambda := \mathrm{sort}\left(\mathrm{eigenvals}\left(M^{-1}\cdot K\right)\right) \qquad \lambda = \begin{pmatrix} 228.015 \\ 1.355\times 10^{3} \\ 2.321\times 10^{3} \\ 3.353\times 10^{3} \\ 6.786\times 10^{3} \end{pmatrix}$$

Natural Frequencies are $\sqrt{\lambda} = \begin{pmatrix} 15.1 \\ 36.812 \\ 48.174 \\ 57.905 \\ 82.38 \end{pmatrix}$ rad/s — 1^{st}, 2^{nd}, 3^{rd}, 4^{th}, 5^{th}

$$U := \mathrm{eigenvecs}\left(M^{-1}\cdot K\right) \qquad U = \begin{pmatrix} -0.323 & -0.591 & 0.625 & 0.152 & -0.324 \\ 0.914 & 0.049 & 0.465 & 0.368 & -0.491 \\ -0.238 & 0.58 & -0.083 & 0.537 & -0.399 \\ 0.052 & -0.491 & -0.395 & 0.584 & 0.101 \\ -9.747\times 10^{-3} & 0.265 & 0.481 & 0.459 & 0.696 \end{pmatrix}$$

These are the eigenvectors or the mode shapes

$i := 0..4$

1^{st}, 2^{nd}, 3^{rd}, 4^{th}, 5^{th}

$$U^{\langle i\rangle} := \mathrm{eigenvec}\left(M^{-1}\cdot K, \lambda_i\right) \qquad U = \begin{pmatrix} 0.152 & -0.324 & 0.625 & -0.591 & -0.323 \\ 0.368 & -0.491 & 0.465 & 0.049 & 0.914 \\ 0.537 & -0.399 & -0.083 & 0.58 & -0.238 \\ 0.584 & 0.101 & -0.395 & -0.491 & 0.052 \\ 0.459 & 0.696 & 0.481 & 0.265 & -9.747\times 10^{-3} \end{pmatrix}$$

Programing in Mathcad

$$I(f,N,a,b) := \left| \begin{array}{l} sum \leftarrow 0 \\ h \leftarrow \dfrac{b-a}{N} \\ \text{for } i \in 1..N-1 \\ \quad sum \leftarrow sum + 2 \cdot f(i \cdot h) \\ sum \leftarrow sum + f(a) + f(b) \\ sum \cdot \dfrac{h}{2} \end{array} \right.$$

A program which calculates the integral of a function between points a and b by using trapezoidal rule.

Always use the "Programing Toolbar to insert programing keywords such as: "for, while, if, otherwise, break, return, continue, on error". writing them will result in error.

The last line will give the output value of a function. If you would like something else to be the output you also use "return" keyword which may be helpful in debugging your code

$$f(x) := e^{x}$$

$I(f,10,0,\pi) = 22.322494$ Program Result

$\int_0^{\pi} f(x)\,dx = 22.140693$ Numerical Integration of Mathcad

NOTE: The values defined in a function will only be known in the function and they cannot be called in the Mathcad file outside of function block. However, any value defined above and outside of the function can be used in the function.