

Message Scheduling for the FlexRay Protocol: The Dynamic Segment

Ece Guran Schmidt and Klaus Schmidt

Abstract—The FlexRay communication protocol is expected to be the *de facto* standard for high-speed, in-vehicle communication. In this paper, we formally investigate the scheduling problem for the *dynamic segment* (DS) of FlexRay. We take the bounds on the generation times and the timing requirements of the signals into consideration to propose a reservation-based scheduling approach that preserves the flexible medium access of the DS. To obtain efficient schedules, we formulate a nonlinear integer programming problem (NIP) that minimizes the required duration of the DS. This NIP is then decomposed into two linear binary integer programming problems to facilitate the computation of feasible message schedules. An experimental study illustrates our message scheduling approach for the DS of FlexRay.

Index Terms—FlexRay, integer programming, real time, scheduling, vehicular communication networks.

NOMENCLATURE

Notation	Explanation
T_c	FlexRay cycle duration.
T_{SS}, T_{DS}	Static segment, dynamic segment duration).
$\tau_{\text{bit}}, T_{MS}$	bit time, minislot duration.
N_{DS}	Number of minislots in the dynamic segment.
\mathcal{N}	Set of nodes on the FlexRay network.
\mathcal{M}_S^n	Sporadic messages of node $n \in \mathcal{N}$.
\mathcal{M}_S	All sporadic messages.
M_m^n	Message m of node n .
pm_m^n, dm_m^n	Period and deadline of M_m^n .
lm_m^n	Length of M_m^n .
$R = (n, rp, w, l)$	Reservation for node n .
rp, w, l	Reservation period, offset, length.
$\mathcal{R}^n, \mathcal{R}$	Set of reservations for node n , all reservations.
$r : \mathcal{M}_S \rightarrow \mathcal{R}$	Map of messages to reservations.
L_j	Cycle load of a FlexRay cycle j .
\mathcal{R}_j	All reserved DYS on FC j .
RP	least common multiple (lcm) of all reservations periods.

$f^n : \mathcal{R}^n \rightarrow \mathcal{F}^n$	Map of reservations to FIDs.
$B^n = \sum_{R \in \mathcal{R}^n} (l/rp)$	Bandwidth reservation for node n per FC.
$B = \sum_{n=1}^N B^n$	Bandwidth reservation for all nodes.
$\mathcal{G}_q^n \subseteq \mathcal{M}_S^n$	Message group q for node n .
$g^n(\mathcal{G}_q^n)$	Reservation for \mathcal{G}_q^n .
$\mathcal{G} = \cup_{n=1}^N \mathcal{G}^n$	All message groups.
$\mathcal{G}_S \subseteq \mathcal{G}$	Selected groups.

I. INTRODUCTION

MECHANICAL and hydraulic components in vehicles have been replaced by electronic components since the 1970s. These in-vehicle electronic systems employ *electronic control units* (ECUs), which are embedded systems with, e.g., a microcontroller, sensors, and actuators. Communication networks enable the information exchange among ECUs to support most of their tasks. These days, more than 70 ECUs exchange around 2500 signals in luxury cars [1], [2].

The communication networks in vehicles transmit signal data that are encapsulated in *messages*. Most of these messages are real-time messages, i.e., their timely delivery must be guaranteed. Technically, precomputed *message schedules* have to be supplied to meet such timing requirements. In addition, considering the fast growth in the number of ECUs and signals in automotive electronics, the communication must be efficient to provide system extensibility.

One of the first in-vehicle communication networks for automotive systems is the controller area network (CAN) [2], [3]. It can provide bounded delay communication among ECUs at data rates between 125 kb/s and 1 Mb/s and is currently the most widely used in-vehicle network. However, it is not suitable for novel applications such as electronic components of the power train or x-by-wire applications, which are hard real time in nature, and require high-speed, robust, and predictable communication. The first attempts to meet these demands are time-triggered CAN (TTCAN [4]), time-triggered protocol (TTP [5], [6]), and ByteFlight [7]. TTCAN and TTP are *time-triggered* technologies with predictable medium access, whereas ByteFlight is based on flexible time-division multiple access (FTDMA), which aims at efficient bandwidth use.

FlexRay in-vehicle communication networks was founded as an industry consortium by BMW, Daimler-Chrysler, Philips, and Freescale in 2000 [8], [9]. Currently, there are more than 150 members in the consortium, and the first series production car with FlexRay was on the road in 2006. FlexRay has a static segment (SS) with time-division multiple access (TDMA)

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E. G. Schmidt is with the Department of Electrical and Electronics Engineering, Middle East Technical University, Ankara 06531, Turkey (e-mail: eguran@metu.edu.tr).

K. Schmidt is with the University of Erlangen-Nuremberg, 91056 Erlangen, Germany (e-mail: klaus.schmidt@rt.eei.uni-erlangen.de).

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operation and a dynamic segment (DS) with FTDMA operation. It is expected to be the new *de facto* standard, combining the advantages of time-triggered and FTDMA communication [2]. It provides two channels with a bandwidth of 10 Mb/s each, enabling applications that were not realizable with CAN, such as x-by-wire.

The DS of FlexRay is designed to accommodate *sporadic* real-time messages that are generated by event occurrences and have to be transmitted before their *deadline*. To this end, it is required to find feasible message schedules that meet the timing requirements. Previous work on the FlexRay DS mostly provides methods to test if a given schedule is feasible [10], [11]. In addition, one study [12] analyzes and evaluates deadline monotonic scheduling.

Different from the previous work, we propose a method for synthesizing efficient and feasible message schedules. Based on a formal problem description, our approach determines the required system parameters such that the sporadic messages are delivered on time. Adopting ideas from our work in [13], we consider the bounds on the message generation times and the timing requirements for message delivery of the sporadic messages to reserve bandwidth for each message while maintaining the benefits of the FTDMA operation of the FlexRay DS. In our framework, we define appropriate performance metrics to measure the efficiency of each schedule. Then, integer programming is applied to select the most efficient feasible schedule.

The structure of this paper is given as follows. In Section II, we describe the operation of FlexRay and introduce our notation for the system parameters. Section III addresses different issues related to message scheduling for the DS. Our idea of message grouping to reduce the bandwidth reservation is first discussed in Section IV and is then employed in Section V to find optimal message schedules. Section VI presents experimental results. Conclusions are given in Section VII.

II. FLEXRAY PROTOCOL

The FlexRay protocol defines two channels that operate at a bandwidth of $C = 10$ Mb/s, each leading to a bit time of $\tau_{\text{bit}} = 0.1 \mu\text{s}$. In this paper, we consider message transmissions on one FlexRay channel.

A. Description of the FlexRay Cycle (FC)

The operation of each FlexRay channel is based on a fixed-duration repeatedly executed FC that is time slotted [8]. The FC comprises an *SS*, a *DS*, a *symbol window* (SW), and the *network idle time* (NIT). The SW and the NIT provide time for the transmission of internal control information and protocol-related computations. The duration of the FC, SS, and DS, which are fixed during the configuration of a given system, are measured in milliseconds and are denoted as T_c , T_{SS} , and T_{DS} , respectively. A generic FC is depicted in the upper part of Fig. 1.

The SS is similar to TTP [6] and employs the TDMA approach. The investigation of the SS is not in the scope of this paper and can be studied as an independent scheduling problem.

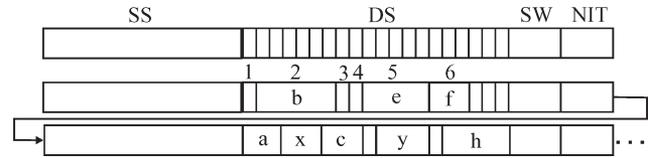


Fig. 1. FlexRay cycle description.

We refer the reader to the companion paper [14] for a detailed description. The DS is similar to ByteFlight [7] and employs the FTDMA approach. The smallest time unit in the DS is the *minislot* (MS) with a duration of T_{MS} (in milliseconds), and the DS contains a fixed number of N_{DS} MS, where $N_{\text{DS}} \leq N_{\text{DS,max}} = 7994$. The DS consists of consecutive *dynamic slots* (DYS) that are superimposed on MS. If a message is transmitted in a DYS, then the length of the DYS is equal to the number of MS needed for message transmission. Otherwise, the length of the DYS is one MS.

Each node maintains a *slot counter* to follow the progress of the DS. It is initialized to 1 at the beginning of each FC and is incremented in every DYS. The arbitration procedure ensures that only frames with a frame ID (FID) that equals the current value of the slot counter can be transmitted [8]. Therefore, we interchangeably use the notion FID to express the frame ID and the value of the slot counter in the remainder of this paper. The DS in Fig. 1 consists of 20 MS. In the first FC, messages are transmitted in the second, fifth, and sixth DYS, whereas the length of, e.g., the second DYS is 6 MS.

B. Messages

We consider a communication system that consists of N nodes (ECUs) that are connected by FlexRay, where the *set of nodes* is $\mathcal{N} = \{1, \dots, N\}$. The nodes exchange *periodic* and *sporadic* real-time messages that are transmitted in *FlexRay Frames*. We assume that all periodic messages are scheduled in the SS as studied in [14]. In this paper, we investigate the transmission of sporadic messages in the DS.

Our representation of the timing properties of sporadic messages follows the lines of related work in [12], [13], and [15]–[17]. For each sporadic message, there is a *deadline*, which is the largest tolerable time interval between the generation and the transmission of the message. In our work, the deadline includes the message transmission time as well as the maximum jitter of the message as defined in [10]. In addition, the recurrence of a sporadic message is described by its minimum interarrival time denoted as *period*, which characterizes the minimum time interval between two consecutive message generations.

The sporadic messages of a node $n \in \mathcal{N}$ constitute a set $\mathcal{M}_S^n = \{M_1^n, \dots, M_{S_n}^n\}$, and the entire set of sporadic messages is denoted as $\mathcal{M}_S := \bigcup_{n=1}^N \mathcal{M}_S^n$. Each sporadic message $M_m^n \in \mathcal{M}_S^n$ has a period pm_m^n and a deadline dm_m^n , where $dm_m^n \leq pm_m^n$. The *length* lm_m^n (in MS) of M_m^n can be computed as in [8], including the signal data s_m^n in multiples of two-byte words, the FlexRay framing overhead $s_m^n \cdot 4$ bits + O_F , and the communication-free DYS idle phase as

$$l_m^n = \lceil (s_m^n \cdot 16 \text{ bits} + s_m^n \cdot 4 \text{ bits} + O_F) \tau_{\text{bit}} / T_{\text{MS}} \rceil. \quad (1)$$

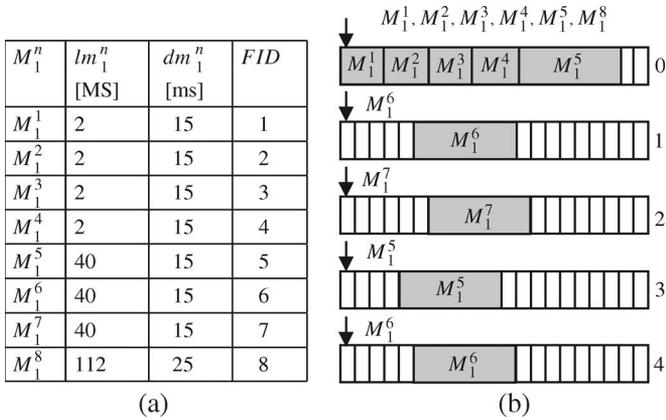


Fig. 2. (a) Message set. (b) DM scheduling.

C. DS Scheduling: Issues and Previous Work

The construction of the FC requires the offline computation of several system parameters. The FC duration T_c has to be chosen considering the bandwidth and delay requirements of the messages that are scheduled in both the FlexRay SS and DS, and the durations T_{SS} of the SS and T_{DS} of the DS have to fulfill $T_{SS} + T_{DS} \leq T_c$. Hence, it is desirable to keep T_{DS} as small as possible to accommodate the bounds given by the SS and the FC time, which depend on the message properties in the SS and the DS. Furthermore, a unique assignment of FIDs to nodes has to guarantee that the messages are transmitted before their deadlines.

Since FlexRay is fairly new compared to the legacy bus standards such as CAN, the literature on the DS is limited. In particular, most approaches assume that the above parameters have already been determined and suggest a schedulability analysis to verify if all message deadlines are met. Cena and Valenzano [18] performed a basic analysis of FTDMA (based on ByteFlight) and provided guidelines to find the maximum FID such that each message is scheduled on the bus cycle from which it arrives. Pop *et al.* [10] conducted a schedulability analysis for the DS, given the above parameters. As an extension of that work, Hagiescu *et al.* [11] modeled the DS using service curves and investigated service bounds for messages. The synthesis of feasible schedules is first addressed in [12], where a deadline-monotonic (DM) approach for assigning FIDs to messages is proposed to compute T_{DS} . In this paper, the response times of the sporadic messages are minimized since feasible schedules are achieved if all response times are smaller than the respective message deadlines. Our approach directly synthesizes feasible schedules that minimize T_{DS} by employing the knowledge about the message deadlines.

We provide a simple example to demonstrate drawbacks related to the DM assignment of FIDs to messages.

Example 1: Let the nodes in $\mathcal{N} = \{1, \dots, 8\}$ communicate using FlexRay with $T_c = 5$ ms, and let the message set $\mathcal{M}_S = \{M_1^1, \dots, M_1^8\}$ in Fig. 2(a) be given. Here, we require that $pm_1^n = dm_1^n$ for $n = 1, \dots, 8$. Furthermore, it is given that the length of the DS is limited to $N_{DS} = 120$ MS.

Assume that all messages arrive at the beginning of the DS. Consider the arrival of messages in Fig. 2(b). Although M_1^8 arrives in FC 0, it cannot be transmitted before FC 4 and, hence,

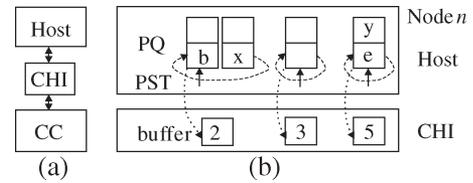


Fig. 3. FlexRay software architecture.

misses its deadline. This is due to the fact that because of the repeated arrival of M_1^5 , M_1^6 , and M_1^7 , there is always a message with a smaller FID to be transmitted, while the remaining DS is shorter than lm_1^8 . Note that this arrival scenario can be extended such that M_1^8 is not scheduled indefinitely.

The drawback of DM scheduling is that each message is transmitted in the first DYS where it fits: When M_1^6 arrives (FC 4), there are two more FCs until its deadline (FC 6). However, M_1^6 is scheduled in FC 4, and M_1^8 misses its deadline.

In this paper, we propose a scheduling policy that tackles this problem. We reserve bandwidth to provide DYS with certain lengths and periodic recurrence. Then, messages can be assigned to these DYS such that they are only transmitted in their reserved DYS. Based on this reservation idea, our scheduling approach determines feasible schedules such that a guaranteed opportunity exists for each message to be transmitted before its deadline while the duration T_{DS} of the DS is minimized and the duration T_c of the FC is chosen, respecting the joint requirements of the SS and the DS, as described in Section III-C.

It is important to note that the FTDMA structure of the DS is preserved. If the messages to be transmitted in a reserved DYS are not ready at the time of the DYS, then the DYS is only $1 T_{MS}$ long, and the next DYS can start immediately.

D. Software Architecture

As specified in [8], the components of each node are a host and a communication controller (CC) that are connected by a controller-host interface (CHI) as shown in Fig. 3(a). In our framework, the host provides the scheduling functionality for the sporadic messages in the DS, while the CC independently implements the FlexRay protocol services. Hence, the FIDs allocated to the nodes do not directly indicate the frames to be sent but indicate the nodes to transmit in a given DYS.

To support reservations, as described above, the CHI of each node contains a buffer for each related FID [see Fig. 3(b)]. The host implements a periodic scheduling table (PST) per allocated FID that indicates the FCs in which the FID is to be used for message transmission. For each used entry in such a PST, the host maintains a corresponding priority queue (PQ) that holds the messages to be sent with the respective FID and FC sorted by increasing deadline. Then, in each reserved DYS, the highest priority sporadic message, i.e., the message with the smallest deadline in the PQ associated to the current FC is assigned to the buffer of the respective FID in the CHI.¹

¹Here, we assume that ties among messages with the same deadline are resolved according to a predetermined rule.

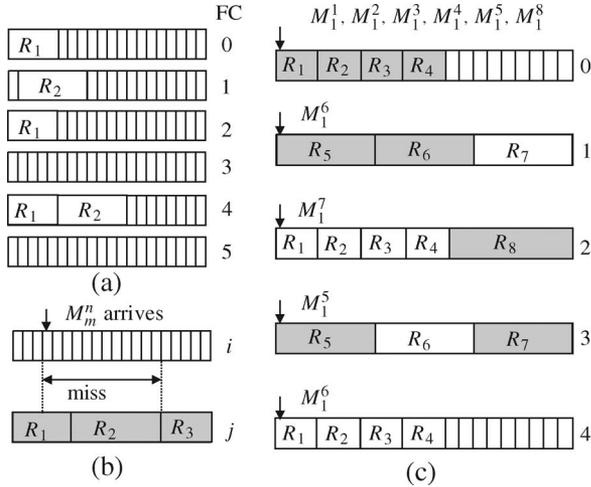


Fig. 4. (a) DYS reservation. (b) Worst-case deadline miss. (c) Example 1.

Fig. 3 shows the software architecture for a FlexRay node that is configured to schedule sporadic messages with FIDs 2, 3, and 5 (compare Fig. 1). In the PQ, the priority increases toward the lower part of the figure. The PST for FID 2 chooses messages from two PQs in alternate FCs, while the PSTs for FIDs 3 and 5 provide access for messages in one PQ in each FC. In the first FC, the sporadic messages b and e are assigned to DYS 2 and 5, as indicated by the solid arrows, while no message is present for the DYS 3. The message y with a lower priority than e has to wait until e is transmitted, while x can be transmitted with FID 2 in the next FC according to its location in the PST.

III. MESSAGE SCHEDULE FOR THE DS

A. Scheduling Issues for the DS

In our scheduling policy, each message M_m^n is mapped to a specific DYS, which has at least lm_m^n reserved MS at least every time period of dm_m^n . Considering that on the one hand, M_m^n can be generated during the dm_m^n interval only once ($dm_m^n \leq pm_m^n$), and on the other hand, the DYS reoccurs before the deadline of M_m^n , it is sufficient to transmit M_m^n during the reserved DYS such that M_m^n meets the deadline. In the following, we formalize this idea.

A reservation R for a node n is a 4-tuple (n, rp, w, l) with the reservation period $rp \in \mathbb{N}$, the offset $w \in \{0, \dots, rp - 1\}$ and the reservation length $l \in \mathbb{N}$. In our scheduling framework, R stands for l MS that are reserved at all FCs $(z \cdot rp + w)$, $z \in \mathbb{N}_0$, while 1 MS is reserved in the remaining FCs. Hence, the bandwidth reservation per FC for a given R is $B_R = l/rp$ MS. Two example reservations for a node n are depicted in Fig. 4(a), where $R_1 = (n, 2, 0, 5)$, and $R_2 = (n, 3, 1, 7)$. The respective bandwidth reservation per FC evaluates to $B_{R_1} = 5/2$ MS and $B_{R_2} = 7/3$ MS.

Each reservation for a node n provides a recurring DYS for at least one message in \mathcal{M}_S^n . Denoting the set of all reservations for n as \mathcal{R}^n and the overall set of reservations as $\mathcal{R} := \bigcup_{n=1}^N \mathcal{R}^n$, this assignment is expressed by the map $r : \mathcal{M}_S \rightarrow \mathcal{R}$, i.e., we require that there is only one reservation for

each message in \mathcal{M}_S . Furthermore, if $r(M_m^n) = (n, rp, w, l)$ for $M_m^n \in \mathcal{M}_S^n$, then $rp \cdot T_c \leq dm_m^n$ and $l \geq lm_m^n$ must be satisfied such that the corresponding DYS can accommodate the length of M_m^n and M_m^n meets its deadline. Together, our goal is to determine \mathcal{R} and r while optimizing the performance metrics defined in Section III-D.

B. FID Assignment

To relate the reservation idea to the software architecture in Section II-D, we note that each reservation has to be associated to an FID and entries in the corresponding PST with their respective PQs. In Section V, we present an optimization approach that guarantees that every FID assignment that obeys the following rules can be used to schedule the sporadic messages.

Let \mathcal{F}^n be the set of FIDs assigned to each node n . The map $f^n : \mathcal{R}^n \rightarrow \mathcal{F}^n$ that relates each reservation to an FID has to fulfill the following conditions.

- 1) FIDs are uniquely assigned to nodes as stated in [8].
- 2) FID 1 is assigned.
- 3) FID assignments are consecutive.
- 4) The total number of FIDs that are assigned to N nodes on a given FlexRay DS is smaller than $N_{DS, \max}$.

As long as f^n satisfies 1)–4), FID assignments are arbitrary for our scheduling policy.

C. Choice of the FC Time

There are constraints for choosing T_c such that feasible schedules can be constructed for any given message set. If a given message M_m^n is restricted to be transmitted in the DS, as discussed in Section II-B, and if T_c is chosen larger than dm_m^n , transmitting M_m^n multiple times in the same FC does not guarantee that M_m^n meets the deadline. The interval between the last transmission of M_m^n in the DS of the previous FC and the first transmission of M_m^n in the DS of the current FC can be longer than dm_m^n . Hence, it must hold that $T_c \leq dm_{\min}$, where dm_{\min} is the minimum deadline among all sporadic message deadlines.

An additional constraint is illustrated in Fig. 4(b), where shaded areas indicate the transmitted messages. Assume that $r(M_m^n) = R_3$ and $rp_3 = dm_m^n$. Let R_1, R_2 , and $R_3 \in \mathcal{R}$, with $f^n(R_1) = 1$, $f^n(R_2) = 2$, and $f^n(R_3) = 3$. In the worst case, M_m^n arrives at MS 3 of FC i and misses its reserved DYS by 1 MS. M_m^n is then transmitted in the next FC j with the reserved DYS for M_m^n , i.e., $j = i + rp_3$. Suppose R_1 and R_2 take up the indicated number of MS in FC j , as shown in Fig. 4(b). As $f^n(R_1) < f^n(R_3)$ and $f^n(R_2) < f^n(R_3)$, the scheduling of M_m^n is delayed in FC j , and M_m^n misses the deadline. This deadline miss is less than T_c , as it is due to the relative MS positions between arrival and transmission of M_m^n in FC i and j , respectively. This can be prevented independent of the FID assignments if $(rp_3 + 1) \cdot T_c \leq dm_m^n$. Following this concept, we denote the required message reservation period for a message M_m^n by $rpm_m^n := dm_m^n/T_c - 1$.

Having determined the message reservation period for each message in \mathcal{M}_S , a good choice for the FC time T_c is the greatest

common divisor (gcd) of all message reservation periods since this choice enables reservations with the maximum allowable scheduling periods. We denote this parameter as $T_{c,DS}$. Furthermore, our result for the SS in [14] indicates that T_c must be an integer divisor of a parameter $T_{c,SS}$. Hence, we propose the use of the FC time $T_c = \text{gcd}(T_{c,SS}, T_{c,DS})$.

For notational convenience, we express dm_m^n , pm_m^n , rpm_m^n , and rp_i in the units of T_c for the rest of this paper. In particular, we now have $rpm_m^n = dm_m^n - 1$, i.e., it must be satisfied for a feasible schedule that if $r(M_m^n) = R$, then $rp \leq rpm_m^n$.

We revisit Example 1 in Fig. 4(c), where our reservation-based scheduling is applied. Here, each message has $r(M_1^i) = R_i$ and $f(M_1^i) = i - 1$, with $rp_i = dm_1^i - 1$ and $i = 1, \dots, 8$. The shaded areas show the reservations that are used to transmit the messages with the arrival pattern that is analogous to Example 1. In our approach, M_1^8 has a reserved DYS to be transmitted every $rp_8 = 5 - 1 = 4$ FCs. Although M_1^7 with a smaller FID arrives in FC 2, it waits for its next reserved DYS in FC 3. Thus, M_1^8 is transmitted before its deadline in the reserved DYS in FC 2, and schedulability is achieved with $N_{DS} = 120$ MS.

D. Performance Metrics

We introduce the *cycle load* L_j of an FC j as the first performance metric. It denotes the maximum number of MS that is reserved for message transmission in FC j for an arbitrary assignment of FIDs, considering that at most one FID can be assigned per message. L_j includes both the case where no message is transmitted for an FID (duration of 1 MS) and the case where a message is transmitted. Let $\mathcal{R}_j \subseteq \mathcal{R}$ be the set of all reserved DYS for message transmission on FC j , i.e., $\forall R \in \mathcal{R}_j, \exists z \in \mathbb{N}_0$ such that $j = (z \cdot rp + w)$. Then, L_j is defined as follows:²

$$L_j = \sum_{R \in \mathcal{R}_j} l + \left(|\mathcal{M}_S| - \sum_{R \in \mathcal{R}_j} 1 \right) = \sum_{R \in \mathcal{R}_j} (l-1) + |\mathcal{M}_S|. \quad (2)$$

As the reservations are periodic, the reservation pattern of the DS repeats every rp FCs, where the *DS reservation period* rp is the lcm of all reservation periods. Hence, the FC with the maximum cycle load occurs within rp FCs. We define the *maximum cycle load* as $L_{\max} = \max_{j \in \{1, \dots, rp\}} (L_j)$. Then, we choose $N_{DS} = L_{\max}$ and minimize L_{\max} in Section V-B to determine a feasible schedule with the shortest possible DS.

The *bandwidth reservation* $B^n = \sum_{R \in \mathcal{R}^n} (l/rp)$ and $B = \sum_{n=1}^N B^n$ indicate the number of MS reserved per FC for each node $n \in \mathcal{N}$ and for all of the nodes, respectively.

The reserved MSs are dedicated to transmitting specific messages, and they cannot be used to transmit new messages. Hence, a low B for a given message set indicates the efficiency of the bandwidth reservation and the extensibility of the system. We propose to minimize B in Section V-C.

²Here, $|\mathcal{M}_S|$ denotes the number of messages in \mathcal{M}_S .

TABLE I
SPORADIC MESSAGES FOR EXAMPLE 2

M_m^n	lm_m^n [MS]	rpm_m^n [T_c]	pm_m^n [T_c]	M_m^n	lm_m^n [MS]	rpm_m^n [T_c]	pm_m^n [T_c]
M_1^n	40	2	4	M_4^n	36	18	20
M_2^n	100	4	6	M_5^n	20	20	40
M_3^n	48	8	8				

IV. MESSAGE GROUPING

A. Message Grouping: Example

In our setting, the relevant timing properties of each message M_m^n are given as the deadline dm_m^n and the period pm_m^n , where, usually, $dm_m^n < pm_m^n$. This means that not necessarily all reservations for M_m^n are utilized to transmit a message M_m^n . Similar to our work in [4], we propose to assign multiple messages to the same reservation while preserving schedulability. As a result, more reservations are utilized such that the bandwidth reservation B is minimized.

The following example message set for a node $n \in \mathcal{N}$ demonstrates this approach. The properties of the sporadic message set $\mathcal{M}_S^n = \{M_1^n, \dots, M_5^n\}$ are listed in Table I.

Example 2: Assume that the messages M_1^n and M_3^n in Table I have the reservations $R_1 = r(M_1^n) = (n, 2, w_1, 40)$ and $R_3 = r(M_3^n) = (n, 8, w_3, 48)$. In this case, a number of $(40/2) + (48/8) = 26$ MS is reserved for R_1 and R_3 per FC. Furthermore, depending on the choice of the offsets w_1 and w_3 , it holds that either $L_{\max} = 48$ MS or that $L_{\max} = 88$ MS. Here, at least $\lceil 8/rpm_1^n \rceil = 4$ DYS *have to be reserved* for M_1^n within eight FCs to guarantee its timely transmission. However, at most $\lceil 8/pm_1^n \rceil = 2$ messages *can be generated*. Hence, at least two reserved DYS for M_1^n remain unused.

Alternatively, we can assign M_1^n and M_3^n to the same reservation R_1 such that $r(M_3^n) = r(M_1^n) = R_1 = (n, \min(rpm_1^n, rpm_3^n), w_1, \max(lm_1^n, lm_3^n)) = (n, 2, w_1, 48)$. For every occurrence of R_1 , only one message is transmitted. If M_1^n and M_3^n are ready at the same time, we always give M_1^n the higher priority as $rpm_1^n < rpm_3^n$. Thus, M_1^n is transmitted according to the software architecture in Section II-D. Nevertheless, the transmission of M_3^n within $rpm_3^n = 8$ FCs after it is generated is guaranteed because M_1^n can only be generated twice during eight FCs, leaving out at least two reserved but unused DYS for the transmission of M_3^n .

The benefits regarding the performance metrics defined in Section III-D are given as follows. When assigning M_1^n and M_3^n to the same reservation R_1 , their contribution to the bandwidth reservation B^n decreases to $\max(lm_1^n, lm_3^n) / \min(rpm_1^n, rpm_3^n) = 24$ MS/ T_c . The maximum cycle load for R_1 now is $L_{\max} = \max(lm_1^n, lm_3^n) = 48$ MS. Consequently, such *grouping* can, indeed, be used to improve the performance metrics as defined in Section III-D.

It can readily be observed that there is more than one assignment of reservations that includes M_1^n . For example, it is possible to assign M_2^n and M_1^n to the same reservation as one DYS for M_1^n remains unused in $rpm_2^n = 4$ FCs. However, not all of such multiple assignments improve our performance metrics. Using separate reservations for M_1^n and M_2^n , the bandwidth reservation is $(lm_1^n / rpm_1^n) +$

$(lm_2^n/rpm_2^n) = 45 \text{ MS}/T_c$, whereas when assigning them to the same reservation, the total bandwidth reservation becomes $\max(lm_1^n, lm_2^n)/\min(rpm_1^n, rpm_2^n) = 50 \text{ MS}/T_c$, which increases B^n . Accordingly, such groups will be excluded in the optimization in Section IV-B.

B. Message Grouping: General Formulation

In this section, the construction of *message groups*, i.e., assignments of (multiple) messages to a reservation for a node $n \in \mathcal{N}$ is generalized. A message group $\mathcal{G}_q^n \subseteq \mathcal{M}_S^n$, $q \in \mathbb{N}$ corresponds to the reservation $g^n(\mathcal{G}_q^n) = (n, rp, w, l)$, where $g^n(\mathcal{G}_q^n) = r(M_m^n)$ for all $M_m^n \in \mathcal{G}_q^n$. Here, we determine $rp = \min_{M_m^n \in \mathcal{G}_q^n} (rpm_m^n)$ and $l_q^n = \max_{M_m^n \in \mathcal{G}_q^n} (lm_m^n)$, as discussed in Sections III-C and D.

The construction process of a message group \mathcal{G}_q^n starts with an empty group, i.e., any message M_m^n can be added to \mathcal{G}_q^n . Then, $\mathcal{G}_q^n = \{M_m^n\}$, $g^n(\mathcal{G}_q^n) = (n, rpm_m^n, w, lm_m^n)$. For the further discussion, we define $RM_{q,l}^n$ and $P_{q,l}^n$ as metrics to check if a new message M_l^n can be added to a nonempty \mathcal{G}_q^n . Let $g^n(\mathcal{G}_q^n) = (n, rp, w, l)$.

$RM_{q,l}^n$ denotes the least number of *remaining* DYS that can be used for M_l^n within a time period of rpm_l^n if all messages with smaller scheduling periods in \mathcal{G}_q^n are generated as frequently as possible and scheduled before M_l^n . We have

$$RM_{q,l}^n = \lfloor rpm_l^n / rp \rfloor - \sum_{M_m^n \in \mathcal{G}_q^n, rpm_m^n \leq rpm_l^n} \lceil rpm_l^n / pm_m^n \rceil. \quad (3)$$

If $RM_{q,l}^n \geq 1$, then M_l^n can be scheduled in \mathcal{G}_q^n , together with the already present higher priority messages.

$P_{q,l}^n$ is the *profit* in B^n when adding M_l^n to \mathcal{G}_q^n compared to the case where M_l^n is scheduled separately, i.e.,

$$P_{q,l}^n = (l/rp + lm_l^n/rpm_l^n) - \max(lm_l^n, l)/rp. \quad (4)$$

If $P_{q,l}^n \geq 0$, then adding M_l^n to \mathcal{G}_q^n decreases B^n . Considering (3) and (4), M_m^n fits into \mathcal{G}_q^n if $RM_{q,l}^n \geq 1$ and $P_{q,l}^n \geq 0$.

There are already two groups $\{M_1^n\}$ and $\{M_1^n, M_3^n\}$ that include M_1^n in Example 2. Next, we construct additional groups with M_1^n . Consider $\mathcal{G}_1^n = \{M_1^n\}$ with $g^n(\mathcal{G}_1^n) = (n, 2, w_1, 40)$. We extend the group to $\mathcal{G}_2^n = \{M_1^n, M_3^n\}$, with $g^n(\mathcal{G}_2^n) = (n, 2, w_2, 48)$, as discussed above. If we consider M_4^n , we see that $RM_{2,4}^n = 1$ and $P_{2,4}^n = 2$. A new group is $\mathcal{G}_3^n = \{M_1^n, M_3^n, M_4^n\}$, with $g^n(\mathcal{G}_3^n) = (n, 2, w_3, 48)$. No more new groups can be formed by extending \mathcal{G}_3^n as M_4^n uses the last available DYS. Another possible message to extend \mathcal{G}_2^n is M_5^n , with $RM_{2,5}^n = 2$ and $P_{2,5}^n = 2$. Then, $\mathcal{G}_4^n = \{M_1^n, M_3^n, M_5^n\}$, with $g^n(\mathcal{G}_4^n) = (n, 2, w_4, 48)$.

Similarly, the set of all possible message groups can be determined algorithmically. Algorithm 4.1 checks if M_l^n can be added to a given group \mathcal{G}_q^n while all existing messages in \mathcal{G}_q^n are still transmitted within their scheduling periods. There are three possible results. If the result is *no_fit*, then M_l^n does not fit into \mathcal{G}_q^n . If the result is *last_fit*, then M_l^n is added to \mathcal{G}_q^n , but no other messages can be added. If the result is *more_fit*, then more messages can be added after M_l^n . If M_l^n can be added to \mathcal{G}_q^n , then Algorithm 4.1 generates the update $\mathcal{G}_q^n = \mathcal{G}_q^n \cup \{M_l^n\}$

and adds the new \mathcal{G}_q^n to the set \mathcal{G}^n of all message groups for node n .

Algorithm 4.1 (Check and Add)

Input: $M_l^n, \mathcal{G}_q^n, \mathcal{G}^n$.

Init: result = *more_fit*

if ($P_{q,l}^n < 0$ or $RM_{q,l}^n < 1$)

 result = *no_fit*

else

$\mathcal{G}_q^n := \mathcal{G}_q^n \cup \{M_l^n\}$ and $\mathcal{G}^n = \mathcal{G}^n \cup \mathcal{G}_q^n$

 if ($RM_{q,l}^n = 1$)

 result = *last_fit*

return result

Algorithm 4.2 uses Algorithm 4.1 to enumerate all possible groups $\mathcal{G}_q^n \subseteq \mathcal{M}_S^n$. Here, the ordered *list* LM_S^n of all messages sorted by increasing deadline is used. Two operators **next**(M_l^n) and **last**(LM_S^n) return the message following M_l^n and the last message in LM_S^n , respectively. The comparison $M_k^n < M_l^n$ returns *true* if M_k^n is located before M_l^n in LM_S^n . Before Algorithm 4.2 is run for $M_m^n \in \mathcal{M}_S^n$, \mathcal{G}_q^n , \mathcal{G}^n , and M_c^n are initialized to $\{M_m^n\}$, $\{\{M_m^n\}\}$, and M_m^n , respectively.

Algorithm 4.2 (Group)

Input: $LM_S^n, M_c^n, \mathcal{G}_q^n, \mathcal{G}^n$

(while $M_c^n < \mathbf{last}(LM_S^n)$)

$M_c^n = \mathbf{next}(M_c^n)$

 temp $\mathcal{G}_q^n = \mathcal{G}_q^n$

 result = **Check and Add**($M_c^n, \text{temp}\mathcal{G}_q^n, \mathcal{G}^n$)

 if (result = *more_fit* and $M_c^n \neq \mathbf{last}(LM_S^n)$)

 temp $M_c^n = M_c^n$

Group($LM_S^n, \text{temp}M_c^n, \text{temp}\mathcal{G}_q^n, \mathcal{G}^n$)

The messages in LM_S^n are checked to fit in \mathcal{G}_q^n . Any remaining capacity is indicated when **Check and Add** returns a *more_fit* value. In this case, a new group is formed, which extends \mathcal{G}_q^n with the remaining of the list of messages by running **Group** recursively.

We apply Algorithm 4.2 to our example message set in Table I and construct the message groups for M_1^n with the scheduling period $rpm_1^n = 2$. The ordered list evaluates to $LM_S^n = M_1^n, M_2^n, M_3^n, M_4^n, M_5^n$. The step-by-step evaluation of the algorithm is depicted in Table II.

The entire set \mathcal{G}^n obtained by applying Algorithm 4.2 for the rest of the messages in \mathcal{M}_S^n is

$$\begin{aligned} \mathcal{G}^n = & \{ \{M_1^n\}, \{M_1^n, M_3^n\}, \{M_1^n, M_3^n, M_4^n\}, \\ & \{M_1^n, M_3^n, M_5^n\}, \{M_1^n, M_4^n\}, \{M_1^n, M_4^n, M_5^n\}, \\ & \{M_1^n, M_5^n\}, \{M_2^n\}, \{M_2^n, M_4^n\}, \{M_2^n, M_5^n\}, \{M_3^n\}, \\ & \{M_4^n\}, \{M_5^n\} \}. \end{aligned}$$

The set \mathcal{G}^n has the following characteristics that need to be addressed. First, there are multiple groups that, for example, contain the message M_1^n . However, in the final schedule, exactly one reservation for M_1^n is required. Thus, one out of these groups has to be selected for transmitting M_1^n . Second, for each group $G_q^n \in \mathcal{G}^n$, the offset of the corresponding reservation

TABLE II
MESSAGE GROUP CONSTRUCTION FOR M_1^n IN EXAMPLE 2

Init: $LM_S^n = M_1^n, M_2^n, M_3^n, M_4^n, M_5^n$
Call 0: Group: $M_c^n = M_1^n, \mathcal{G}_q^n = \{M_1^n\}, \mathcal{G}^n = \{\{M_1^n\}\}$
$M_c^n = M_2^n, temp\mathcal{G}_q^n = \{M_1^n\}$ $P_{q,2}^n = (20 + 25) - 50 = -5 \Rightarrow result = no_fit$
$M_c^n = M_3^n, temp\mathcal{G}_q^n = \{M_1^n\}$ $P_{q,3}^n = (20 + 6) - 24 = 2, RM_{q,3}^n = \lfloor 8/2 \rfloor - \lfloor 8/4 \rfloor = 2$ $\Rightarrow result = more_fit$ $temp\mathcal{G}_q^n = \{M_1^n, M_3^n\}, tempM_c^n = M_3^n$
Call 1: Group: $M_c^n = M_3^n, \mathcal{G}_q^n = \{M_1^n, M_3^n\}, \mathcal{G}^n = \{\{M_1^n\}, \{M_1^n, M_3^n\}\}$
$M_c^n = M_4^n, temp\mathcal{G}_q^n = \{M_1^n, M_3^n\}$ $P_{q,4}^n = (24 + 2) - 24 = 2$ $RM_{q,4}^n = \lfloor 18/2 \rfloor - (\lfloor 18/4 \rfloor + \lfloor 18/8 \rfloor) = 1 \Rightarrow result = last_fit$ $temp\mathcal{G}_q^n = \{M_1^n, M_3^n, M_4^n\}$ $\mathcal{G}^n = \{\{M_1^n\}, \{M_1^n, M_3^n\}, \{M_1^n, M_3^n, M_4^n\}\}$
$M_c^n = M_5^n, temp\mathcal{G}_q^n = \{M_1^n, M_3^n\}$ $P_{q,5}^n = (24 + 1) - 24 = 1$ $RM_{q,5}^n = \lfloor 20/2 \rfloor - (\lfloor 20/4 \rfloor + \lfloor 20/8 \rfloor) = 2 \Rightarrow result = more_fit$ $temp\mathcal{G}_q^n = \{M_1^n, M_3^n, M_5^n\}, M_c^n = last(LM_S^n)$ $\mathcal{G}^n = \{\{M_1^n\}, \{M_1^n, M_3^n\}, \{M_1^n, M_3^n, M_4^n\}, \{M_1^n, M_3^n, M_5^n\}\}$
Return 1: Group
$M_c^n = M_4^n, temp\mathcal{G}_q^n = \{M_1^n\}$ $P_{q,4}^n = (20 + 2) - 20 = 2$ $RM_{q,4}^n = \lfloor 18/2 \rfloor - \lfloor 18/4 \rfloor = 4 \Rightarrow result = more_fit$ $temp\mathcal{G}_q^n = \{M_1^n, M_4^n\}, tempM_c^n = M_4^n$
Call 2: Group: $M_c^n = M_4^n, \mathcal{G}_q^n = \{M_1^n, M_4^n\}$ $\mathcal{G}^n = \{\{M_1^n\}, \{M_1^n, M_3^n\}, \{M_1^n, M_3^n, M_4^n\}, \{M_1^n, M_3^n, M_5^n\}, \{M_1^n, M_4^n\}\}$
$M_c^n = M_5^n, temp\mathcal{G}_q^n = \{M_1^n, M_4^n\}$ $P_{q,5}^n = (24 + 1) - 24 = 1$ $RM_{q,5}^n = \lfloor 20/2 \rfloor - (\lfloor 20/4 \rfloor + \lfloor 20/20 \rfloor) = 4 \Rightarrow result = more_fit$ $temp\mathcal{G}_q^n = \{M_1^n, M_4^n, M_5^n\}, M_c^n = last(LM_S^n)$ $\mathcal{G}^n = \{\{M_1^n\}, \{M_1^n, M_3^n\}, \{M_1^n, M_3^n, M_4^n\}, \{M_1^n, M_3^n, M_5^n\}, \{M_1^n, M_4^n\}, \{M_1^n, M_4^n, M_5^n\}\}$
Return 2: Group
$M_c^n = M_5^n, temp\mathcal{G}_q^n = \{M_1^n\}$ $P_{q,5}^n = (20 + 1) - 20 = 1$ $RM_{q,5}^n = \lfloor 20/2 \rfloor - \lfloor 20/4 \rfloor = 5 \Rightarrow result = more_fit$ $temp\mathcal{G}_q^n = \{M_1^n, M_5^n\}, M_c^n = last(LM_S^n)$ $\mathcal{G}^n = \{\{M_1^n\}, \{M_1^n, M_3^n\}, \{M_1^n, M_3^n, M_4^n\}, \{M_1^n, M_3^n, M_5^n\}, \{M_1^n, M_4^n\}, \{M_1^n, M_4^n, M_5^n\}\}$
Return 0: Group

$g^n(\mathcal{G}_q^n)$ has not been determined yet. In Section V, we employ the enumeration of *all* possible message groups, as derived above, to find the choice of groups and reservation offsets that optimizes the performance metrics in Section III-D.

V. OPTIMAL SCHEDULING OF MESSAGES

The *schedule* for \mathcal{M}^S establishes the number of reserved DYS for each $M_m^n \in \mathcal{M}^S$. As we discussed in Section III-A, FIDs can be assigned to reservations arbitrarily. Hence, the parameters left to be determined are the selection of message groups to be used and the offsets for corresponding reservations while minimizing the cycle load.

A message M_m^n can be included in multiple groups in \mathcal{G}^n . Among these groups, one $\mathcal{G}_q^n \in \mathcal{G}^n$, with $M_m^n \in \mathcal{G}_q^n$, must be *selected* such that one and only one reservation for M_m^n is included in the schedule. Let $\mathcal{G} := \bigcup_{n=1}^N \mathcal{G}^n$ denote the set of all groups, and let $\mathcal{G}_S \subseteq \mathcal{G}$ denote the set of *selected groups*. Once \mathcal{G}_q^n is selected, the reservation $(n, rp, w, l) := g^n(\mathcal{G}_q^n)$ is allocated to node n to transmit messages in \mathcal{G}_q^n , with a contribution of l/rp to the bandwidth reservation B^n . Furthermore,

TABLE III
MESSAGE GROUPS FOR EXAMPLE 3

$\mathcal{G}_1 = \{M_1^1\}, pm_1^1 = 3, dm_1^1 = 5$ $R_1 = (1, 2, w_1, 20)$	$\mathcal{G}_2 = \{M_1^1, M_2^1\}$ $R_2 = (1, 2, w_2, 30)$
$\mathcal{G}_3 = \{M_2^1\}, pm_2^1 = 5, dm_2^1 = 7$ $R_3 = (1, 4, w_3, 30)$	$\mathcal{G}_4 = \{M_3^1\}, pm_3^1 = 4, dm_3^1 = 6$ $R_4 = (1, 3, w_4, 10)$
$\mathcal{G}_5 = \{M_1^2\}, pm_1^2 = 3, dm_1^2 = 7$ $R_5 = (2, 2, w_5, 22)$	$\mathcal{G}_6 = \{M_1^2, M_2^2\}$ $R_6 = (2, 2, w_6, 48)$
$\mathcal{G}_7 = \{M_1^2, M_2^2, M_3^2\}$ $R_7 = (2, 2, w_7, 48)$	$\mathcal{G}_8 = \{M_1^2, M_3^2\}$ $R_8 = (2, 2, w_8, 30)$
$\mathcal{G}_9 = \{M_1^2, M_4^2\}$ $R_9 = (2, 2, w_9, 42)$	$\mathcal{G}_{10} = \{M_2^2\}, pm_2^2 = 7, dm_2^2 = 9$ $R_{10} = (2, 6, w_{10}, 48)$
$\mathcal{G}_{11} = \{M_3^2\}, pm_3^2 = 7, dm_3^2 = 9$ $R_{11} = (2, 6, w_{11}, 30)$	$\mathcal{G}_{12} = \{M_4^2\}, pm_4^2 = 5, dm_4^2 = 5$ $R_{12} = (2, 4, w_{12}, 42)$

depending on the offset w , the cycle load of certain FCs is increased by l .

We propose an integer programming approach to determine \mathcal{G}_S and w for the reservation of each $\mathcal{G}_q^n \in \mathcal{G}_S$ such that B and L_{max} are minimized, as discussed in Section III-D.

We illustrate our optimal message scheduling approach with the following example:

Example 3: Let $\mathcal{N} = \{1, 2\}$. We assume that the groups in \mathcal{G} have been computed as listed in Table III by running Algorithm 4.2 on a given set of messages \mathcal{M}_S .

Our goal is to determine \mathcal{G}_S and the offsets w_i of the corresponding reservations such that L_{max} (and, thus, the required duration T_{DS} of the DS) is minimized.

A. Exact Formulation

We formulate integer programming problems with two components to find the optimal message schedule. The first component addresses the selection of the message groups and the corresponding reservations. The binary decision variable $g_i \in \{0, 1\}$ takes the value of 1 if $\mathcal{G}_i \in \mathcal{G}_S$ and is 0 otherwise. The second component is determining the reservation offsets. A reservation R_i can have an offset $w_i \in \{0 \dots rp_i - 1\}$. The binary decision variable $x_{i,k} \in \{0, 1\}$ takes the value of 1 if $w_i = k$ and is 0 otherwise, where $k = 0, \dots, rp_i - 1$. Furthermore, it can readily be observed that the reservation pattern repeats after G_{RP} FCs, where $G_{RP} = \text{lcm}_{\mathcal{G}_i \in \mathcal{G}} rp_i$ is the least common multiple of the reservation periods rp_i corresponding to each group $\mathcal{G}_i \in \mathcal{G}$. Hence, only the FCs $0, \dots, G_{RP} - 1$ need to be taken into account.

Consider a reservation R_i that corresponds to $\mathcal{G}_i \in \mathcal{G}$. The contribution of R_i to L_j , $j = 1, \dots, G_{RP} - 1$, is given as follows.

- 1) $g_i = 0$: Then, $\mathcal{G}_i \notin \mathcal{G}_S$ and R_i does not add to L_j .
- 2) $g_i = 1$ and $x_{i,k} = 1$ for $k = j \bmod rp_i$: Then, $w_i = k$ and l_i MS are reserved for R_i in L_j .
- 3) $g_i = 1$ and $x_{i,k} = 0$ for $k = j \bmod rp_i$: Then, $w_i \neq k$ and one MS is reserved for R_i in L_j .

Accordingly, we can express the cycle load L_j as follows:

$$L_j = \sum_{\mathcal{G}_i \in \mathcal{G}} g_i \cdot (x_{i,k} \cdot l_i + (1 - x_{i,k}) \cdot 1) \quad (5)$$

where $k = j \bmod rp_i$. Any FC $j \in \{0, \dots, G_{RP} - 1\}$ can have the maximum cycle load. Assuming without loss of generality

that $L_{\max} = L_0$, the expression to be minimized is

$$\min_X L_0 = \min_X \sum_{g_i \in \mathcal{G}} g_i \cdot x_{i,0} \cdot l_i + g_i(1 - x_{i,0}) \cdot 1 \quad (6)$$

where X is a vector with all variables g_i and $x_{i,k}$. The requirement that only one reservation is selected for each M_m^n and exactly one offset is determined for each used reservation is formulated by the constraints in (7) and (8), respectively. Since $L_{\max} = L_0$, there are no FCs $j \in \{1, \dots, G_{\text{RP}} - 1\}$, with $L_j > L_0$. This constraint is stated in

$$\forall M_m^n, \quad \sum_{i, M_m^n \in \mathcal{G}_i} g_i = 1 \quad (7)$$

$$\text{for } i = 1, \dots, |\mathcal{G}|, \quad \sum_{k=0}^{rp_i-1} x_{i,k} = g_i \quad (8)$$

$$\text{for } j = 1, \dots, G_{\text{RP}} - 1, \quad L_j \leq L_0. \quad (9)$$

The optimal message schedule is the solution of the optimization problem with the objective function in (6) and the constraints in (7)–(9). Note that it is a nonlinear integer programming problem (NIP), as the computations in (5) and (9) contain products of the decision variables g_i and $x_{i,k}$.

A feasible schedule for Example 3 has been computed using the Tomlab optimization environment [19]. As a result, $g_2 = g_4 = g_9 = g_{10} = g_{11} = 1$ and $g_i = 0$ for the remaining values of i have been found. Hence, $\mathcal{G}_S = \{\{M_1^1, M_2^1\}, \{M_3^1\}, \{M_1^2, M_4^2\}, \{M_2^2\}, \{M_3^2\}\}$. Furthermore, the corresponding offsets are $w_2 = 0, w_4 = 2, w_9 = 1, w_{10} = 0, w_{11} = 1$, and the worst-case maximum cycle load is $L_{\max} = 81$ MS.

Respecting the conditions in Section III-B, the FIDs can now be assigned to the reservations of the selected message groups arbitrarily. However, a further analysis of the NIP solution can reduce the number of required FIDs and, hence, lead to a shorter DS by assigning multiple reservations of one node that never appear in the same FC to the same FID.³

An efficient FID assignment for Example 3 is $f^1(R_2) = 1, f^1(R_4) = 2, f^2(R_9) = 3, f^2(R_{11}) = 4$, and $f^2(R_{10}) = 3$, as shown in Fig. 5(a). Here, the reservations R_9 and R_{10} of node 2 can be assigned to the same FID, instead of choosing a separate FID such as $f^2(R_{10}) = 5$, as they never occupy the same FC. Hence, the two unused MSs that would appear in the longest FC (with R_2 and R_{10}) in the latter case can be eliminated. The resulting cycle load is reduced from 81 to 79 MS. Fig. 5(b) depicts the corresponding software architecture according to Section II-D.

B. Two-Step Formulation

As solving the offline NIP is a hard problem, even for small message sets, we propose to decompose the formulation in (6)–(9) into two linear binary integer programming problems (BIPs) to enable the problem solution also for large message

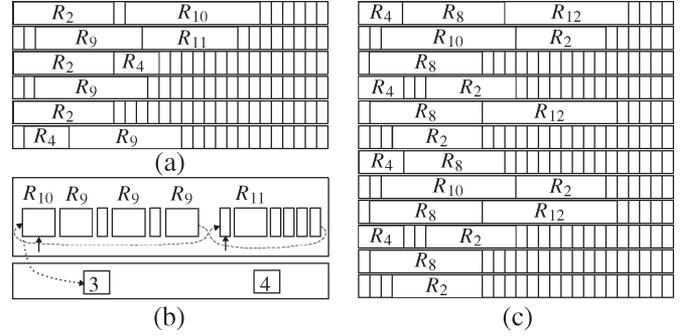


Fig. 5. Optimal reservations for Example 3. (a) NIP. (b) Software architecture. (c) Decomposed BIP.

sets. In the first step, we suggest that the groups to be scheduled are selected such that our performance metric B is minimized. In the second step, the offsets for these selected groups are computed to minimize L_{\max} .

The following objective function is used to minimize B^4 :

$$\min_X B = \min_X \sum_{g_i \in \mathcal{G}} g_i \cdot (l_i / rp_i) \quad (10)$$

subject to the constraints

$$\forall M_m^n \in \mathcal{M}_S, \quad \sum_{i, M_m^n \in \mathcal{G}_i} g_i = 1. \quad (11)$$

Completing the first step yields $\mathcal{G}_S \subseteq \mathcal{G}$, where $\mathcal{G}_i \in \mathcal{G}_S \Leftrightarrow g_i = 1$. In the following step, L_{\max} is minimized. Here, we denote the number of groups in \mathcal{G}_S as G_S and the lcm of their reservation periods as $G_{S, \text{RP}}$. We have

$$L_{\max} = \min_X L_0 = \min_X \sum_{g_i \in \mathcal{G}_S} x_{i,0} \cdot l_i + (1 - x_{i,0}) \cdot 1 \quad (12)$$

subject to the constraints

$$\text{for } j = 1, \dots, G_{S, \text{RP}} - 1 : L_j \leq L_0 \quad (13)$$

$$\text{for } i = 1, \dots, G_S : \sum_{k=0}^{rp_i-1} x_{i,k} = 1. \quad (14)$$

The BIP in (10) and (11) has been solved for Example 3 using Tomlab [19]. As a result, $g_2 = g_4 = g_8 = g_{10} = g_{12} = 1$ and $g_i = 0$ for the remaining values of i . Hence, $\mathcal{G}_S = \{\{M_1^1, M_2^1\}, \{M_3^1\}, \{M_1^2, M_3^2\}, \{M_2^2\}, \{M_4^2\}\}$. Note that, different from the NIP solution, M_3^2 is grouped with M_1^2 in \mathcal{G}_S since the resulting bandwidth reservation $B = 55.1$ MS is smaller than for the NIP solution with $B = 55.7$ MS.

In the next step, we solve the BIP in (12) and (13) for Example 3. $x_{2,1}, x_{4,0}, x_{8,0}, x_{10,1}$, and $x_{12,0}$ are found to be 1, while the rest of the $x_{i,k}$ is 0. The worst-case maximum cycle load is $L_{\max} = 84$ and can be reduced to 82 MS by an efficient FID assignment, as depicted in Fig. 5(c). Although the resulting

³An algorithmic approach to tackle such FID assignment has been developed but is not in the scope of this paper.

⁴Note that the fractional coefficients of the objective function can be converted into integers by multiplying B with G_{RP} .

TABLE IV
SPORADIC MESSAGES OF THE SAE BENCHMARK

sender	1	2	3	5	6	6
# signals	1	8	6	11	4	1
deadline [T_c]/period [T_c]	4/4	4/10	4/10	4/10	4/10	1/10

DS is slightly larger compared to the NIP solution, now only two BIPs have to be solved.

Together, it can be stated that there is a possible tradeoff between the bandwidth reservation B and the maximum cycle load L_{\max} . Furthermore, the decomposed optimization can both provide an upper bound for the minimum L_{\max} and a good initial feasible solution for running the NIP.

VI. EXPERIMENTAL STUDY

In this section, we present a comparison of the NIP solution in Section V-B and the BIP solution in Section V-C. Furthermore, we study the schedule construction for large message sets. In all our experiments, we used the CPLEX solver of Tomlab [19] to obtain the integer programming solutions, and for each data point, ten sample runs have been evaluated on a personal computer with a dual-core Pentium 4 3.4-GHz processor and 1 GB of random access memory.

A. Society of Automotive Engineers (SAE) Benchmark Set

The SAE benchmark set in [15] comprises 31 sporadic messages with data sizes smaller than 8 bits whose deadlines and periods are integer multiples of 5 ms and are transmitted by five nodes (see Table IV). With the choice of $T_{MS} = 6.0 \mu s$ and $O_F = 90$ bits (see [8]), each message fits into a frame with $[(2 \cdot 20 + 90) \cdot 0.1 / 6.0] MS = 3 MS$. For the SAE benchmark set, the NIP and BIP formulations, in combination with the efficient FID assignment, yield the same result of $T_{DS} = 26T_{MS} = 156 \mu s$.

B. Comparison Between the NIP and the BIP Solutions

Our comparison between the solutions of the NIP formulation in Section V-B and the BIP formulation in Section V-C is based on the SAE message set in Table IV that represents a practical message set and the example message set in Table III that was constructed to illustrate the potential different solutions for the NIP and the BIP. As in the SAE message set, we employ five nodes communicating on the FlexRay bus. Since the number of messages in both sets is not sufficient to generate considerable traffic on FlexRay, we extend these sets by randomly choosing messages from the respective set and assigning them to one of five FlexRay nodes until a given arrival rate is reached. Here, for a given message set M_S , we denote the arrival rate as $\sum_{M_m^n \in M_S} l_m^n / dm_m^n$. For both message sets, the NIP can be solved for up to 16 messages in the DS, while the solver fails to find an optimal solution for larger message sets. Fig. 6(a) (SAE messages) and Fig. 6(c) (constructed message set) plot the maximum cycle load L_{\max} , as computed in (12) against the arrival rate. It can be seen that the NIP and BIP formulations yield the same optimization

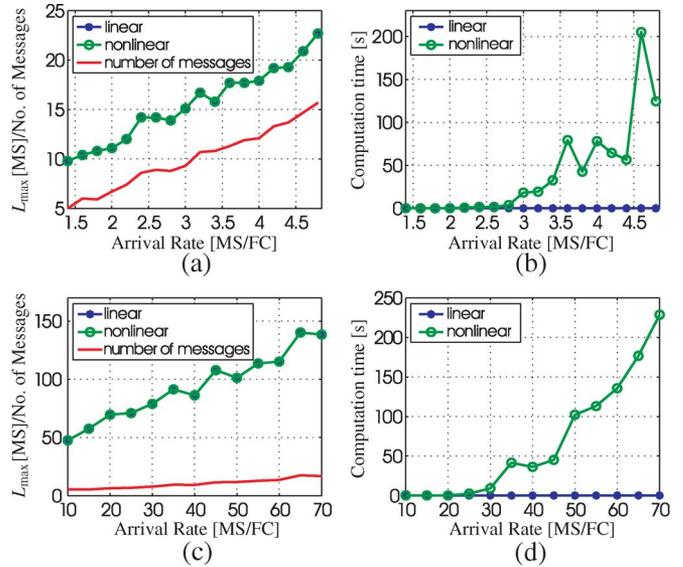


Fig. 6. Cycle load. (a) SAE message set and (b) constructed message set. Optimization time. (c) SAE message set and (d) constructed message set.

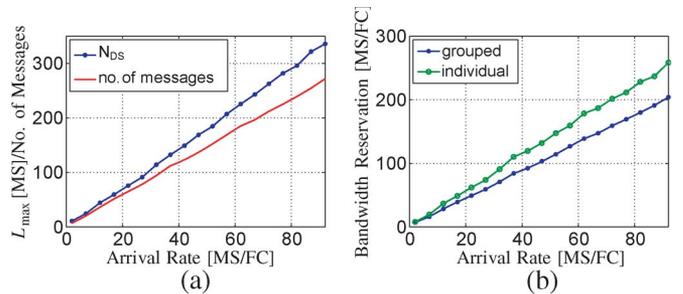


Fig. 7. Large message sets. (a) DS length. (b) Effect of grouping.

results in all our experiments. This suggests that although cases exist where the BIP does not give an optimal solution (compare Example 3), BIP is suitable in practical examples. Moreover, solving the BIP is much less computationally expensive, as illustrated in Fig. 6(b) and (d).

C. Message Scheduling for Large Message Sets

Further experiments were carried out to evaluate the BIP approach for larger message sets. Fig. 7(a) shows that more than 270 messages of the SAE message set can be scheduled in a DS with $T_{DS} \leq 336 \cdot T_{MS} = 2.0 ms$, while computation times of less than 1 h are required. The benefit of the grouping idea is presented in Fig. 7(b) by comparing the bandwidth reservation B [see (10)] needed to schedule individual messages (“individual”) to the bandwidth reservation with grouping (“grouped”). In all cases, the bandwidth required to schedule the given message set is reduced by about 20%.

D. Discussion

It has to be noted that allowing arbitrary FID assignments in the optimization according to Section III-B potentially leads to suboptimal bandwidth reservations. However, it is readily

observed that determining a globally optimal FID assignment is computationally intractable since its computational complexity is even higher than the NIP formulated in Section V-B, which can only be solved for small message sets, as shown in Section VI-B. In this respect, the decomposition of the NIP into two BIP enables the schedule construction for large message sets, as described in Section VI-C, while the experiments in Section VI-B indicate that for practical message sets, the NIP and BIP formulations lead to identical results. Hence, our BIP approach generates feasible schedules for large message sets, while ensuring a minimal bandwidth reservation and the shortest possible duration T_{DS} of the DS.

VII. CONCLUSION

This paper has addressed the message schedule construction for *sporadic* real-time messages that are to be transmitted in the DS of the FlexRay protocol. Our approach proposes to reserve bandwidth such that each sporadic message can meet its *deadline*. Based on a formal description of the scheduling problem, we determine an NIP to compute an optimal message schedule. Here, the bandwidth *reservation* and the *cycle load* are employed as appropriate *performance metrics* that have to be minimized.

To facilitate the problem solution, we suggest a decomposition of the NIP into two BIPs to approximate the optimal result. First, we find a set of reservations that minimizes the bandwidth reservation, and then, we schedule the obtained reservations such that the cycle load is minimized.

The performance of the proposed approach was evaluated in an experimental study. It is verified that the NIP and BIP formulations yield identical results for practical message sets. Furthermore, it was possible to construct feasible schedules for large message sets. Together, our approach enables the algorithmic computation of an optimal schedule for the sporadic messages in the FlexRay DS.

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Ece Guran Schmidt received the B.S. degree in electrical and electronics engineering from the Middle East Technical University, Ankara, Turkey, in 1997 and the M.S. and Ph.D. degrees in electrical and computer engineering from Carnegie Mellon University, Pittsburgh, PA, in 2001 and 2004, respectively.

She is currently a Faculty Member with the Department of Electrical and Electronics Engineering, Middle East Technical University. Her research interests include high-speed networks, networked control systems, and vehicular communication networks.



Klaus Schmidt received the Diploma and Ph.D. (with distinction) degrees in electrical, electronic, and communication engineering from the University of Erlangen-Nuremberg, Erlangen, Germany, in 2002 and 2005, respectively.

He is currently a Postdoctoral Researcher and the Chair of Automatic Control with the University of Erlangen-Nuremberg. His research interests include controller synthesis for discrete event systems, networked control systems, and vehicular communication networks.