A Study on the Performance of a Complementary Auxiliary Antenna Pattern for Maisel Sidelobe Blanker

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Abstract—The problem of coupling between probability of target blanking ($P_{TB}$) and probability of blanking ($P_B$) in Maisel sidelobe blanker (SLB) is addressed and a complementary auxiliary antenna pattern is proposed for phased array radar systems. The numerical results indicate that the complementary pattern provides an improvement on $P_{TB}$ and $P_B$, especially for the cases where antennas have poor mainlobe-to-sidelobe ratio.

Keywords—Sidelobe blanker, phased array radar, radar signal processing.

I. INTRODUCTION

In radar receivers, detected signals are assumed to be due to a target located in the main beam of the antenna. However, a strong echo entering from the sidelobe region of the antenna pattern might also be perceived as a target located in the main beam. This phenomenon causes false alarms and direction finding errors [1]. One way to deal with this problem is to have sufficiently low sidelobe antenna patterns; but, this may be an insufficient solution against stronger echoes. In [2], it was stated that sidelobe blanking (SLB) systems can prevent detections captured by sidelobes. In order to realize that, a two-channel receiver system was proposed [2]: The main channel and the auxiliary channel, in which the pattern of the auxiliary channel is above than the sidelobes of the main channel, system is proposed as shown in Figure 1.

![Fig. 1. Main and auxiliary pattern proposed by Maisel.](image)

The working principle of the proposed system depends on the power relation between the samples of the main and auxiliary channels as in Figure 2 [2]. If the magnitude square of the sample under test in the auxiliary channel is larger than the one in the main channel, it will be gated. On the other hand, if the magnitude square of the sample in the main channel is larger than that in the auxiliary channel, the sample will be further processed.

![Fig. 2. Basic sidelobe blanking system.](image)

This system is indeed simple in operation for the noise free case. However, when it is realized in noisy environment, the overall probability of detection and thereby the system performance might be reduced. To clarify, even if a target appears in the main beam, the magnitude square of the sample in the auxiliary channel can be larger than that in the main channel due to the system noise. This undesired effect of SLB system is called as target blanking. In order to decrease the probability of target blanking, a threshold, $F$, is assigned for the comparison of the main and auxiliary channels.

The main aim of the SLB system is to increase the probability of blanking, $P_B$, and to decrease the probability of target blanking, $P_{TB}$. Considering the main and auxiliary channel patterns given in Figure 1 and assuming a fixed main channel pattern, it is straightforward to understand that improvement on $P_B$ can be achieved by increasing the auxiliary antenna pattern level, whereas reduction on $P_{TB}$ can be handled with decreasing auxiliary antenna pattern level. This phenomenon was also expressed in [2] with the concluding remark of the trade-off between $P_B$ and $P_{TB}$ can be resolved only by the choice of a sufficiently high-gain auxiliary antenna. The conventional SLB system is widely accepted and has been used in many systems; however, to the best of our knowledge, a solution of the coupling between $P_B$ and $P_{TB}$ for systems utilizing low-gain auxiliary antennas is not given in the literature.
Considering the idea of decoupling $P_B$ and $P_{TB}$, we propose a complementary auxiliary antenna pattern in this paper. Numerical results show that SLB system with the proposed auxiliary pattern is advantageous over the conventional structure in terms of $P_B$ and $P_{TB}$. Although the improvement on system performance seems negligible for the antennas with high main-to-sidelobe ratio, the contribution becomes drastic for the particular choices of threshold values and the antennas having poor main-to-sidelobe ratio.

The rest of the paper is organized as follows. In Section II, configuration of the auxiliary antenna is introduced. In Section III, the well known formulas for interferer and target blanking probabilities are revised according to the complementary pattern. In Part IV, performance of the proposed complementary pattern is analyzed and results are demonstrated.

II. PROPOSED AUXILIARY ANTENNA PATTERN

The proposed complementary auxiliary antenna pattern is presented in Figure 3.

In Figure 3, the auxiliary pattern is divided into two parts, namely the main and sidelobe part. Contrary to the classical patterns, the complementary auxiliary pattern has “nulls”, $\omega_m$, in its main part and peaks, $\omega_s$, in its sidelobe part which is analogous to a bandstop filter in space. Note that the aim of the auxiliary antenna is to provide information for sidelobe targets or jammers. In addition, the desired magnitude response of the auxiliary channel against mainlobe targets is to generate small values compared to the main channel. In fact, if an ideal mechanism is to be designed, auxiliary pattern should include zeros in its main part and the maximum gain in its sidelobe part. To approach the ideal case, it is suggested to keep the sidelobe part of the auxiliary pattern higher while reducing pattern of the main part.

When the SLB structure was first proposed in [2] in 1968, the design of the suggested pattern was not feasible. However, such an auxiliary antenna pattern can be designed, relatively easily, with today’s modern phased array radar systems having hundreds of elements. In this paper, the concept of complementary auxiliary antenna is examined and performance of the system is analyzed. The implementation of the complementary auxiliary pattern in a modern phased array radar system is studied in [3]. Besides, it is important to note that a practical approximation to implementation of such an auxiliary pattern can be realized with the difference pattern [4]. Although the implementation of this system is very easy with a monopulse radar, it could be less effective since the difference pattern, in general, does not guarantee higher sidelobes over the main pattern [1].

III. PERFORMANCE EVALUATION CRITERIA

As very well appreciated, the performance of the SLB system can be evaluated by means of the probability of blanking an interference, $P_B$, and blanking a mainlobe target, $P_{TB}$. Probability calculation for SLB systems is examined in detail in [1] and [5]. In this part of the paper, evaluation of SLB systems is modified for the proposed auxiliary pattern design. The performance is analyzed for Swerling 1 target model which assumes the amplitude is Rayleigh distributed and phase is uniformly distributed over $[0, 2\pi]$ [6]. The detailed derivations for the probability of blanking, $P_B$, and the probability of target blanking, $P_{TB}$, are given in [3]. Here a summary of the work conducted in [3] is given.

The performance of the system is evaluated by testing three hypotheses:

- $H_0$: Null hypothesis corresponding to the noise in the channels.
- $H_1$: Target in the mainlobe and no interference, i.e. no target or jammer in sidelobe.
- $H_2$: Interference in the sidelobe and no target in the mainlobe.

In Figure 2, complex valued samples of the main and auxiliary channel signals upstream from the square-law detector are denoted by $S$ and $R$. Considering the three hypothesis and following the notation in [5], $S$ and $R$ can be written as

$$H_0: \begin{cases} S = W_s \\ R = W_r \end{cases}$$

$$H_1: \begin{cases} S = A e^{j\phi_A} + W_s \\ R = \omega_m A e^{j\phi_A} + W_r \end{cases}$$

$$H_2: \begin{cases} S = C e^{j\phi_C} + W_s \\ R = \frac{\omega_s}{2} C e^{j\phi_C} + W_r \end{cases}$$

where $W_s$ and $W_r$ refer to the circular symmetric complex white Gaussian distributed noise samples with zero mean and $\sigma_s^2$ and $\sigma_r^2$ variance, respectively. $\phi_A$ and $\phi_C$ are the phases of the target in the mainlobe and target or jammer in the sidelobe. Both $\phi_A$ and $\phi_C$ are uniformly distributed over $[0, 2\pi]$. $A$ and $C$ are the magnitude of the target in the mainlobe and interference in the sidelobe, respectively. Both $A$ and $C$ are Rayleigh distributed,

$$p_A(c) = \frac{c}{\sigma_A^2} \exp\left(-\frac{c^2}{2\sigma_A^2}\right), \quad c > 0,$$

$$p_C(c) = \frac{\alpha}{\sigma_C^2} \exp\left(-\frac{\alpha^2}{2\sigma_C^2}\right), \quad \alpha > 0,$$

where $\sigma_A^2$ and $\sigma_C^2$ are the average power of the target in the mainlobe and interference in the sidelobe, respectively. $U$ and $V$ denote the samples downstream from the square-law detector of the main and auxiliary channels, respectively. The
distributions of $U$ and $V$ can be given as

$$p_U(u|H_0) = \frac{1}{2\sigma^2} \exp \left( -\frac{u}{2\sigma^2} \right),$$

$$p_U(u|H_1, A) = \frac{1}{2\sigma^2} \exp \left( -\frac{u + \omega_m a^2}{2\sigma^2} \right) I_0 \left( \frac{\omega_m \sqrt{u}}{\sigma^2} \right),$$

$$p_U(u|H_2, C) = \frac{1}{2\sigma^2} \exp \left( -\frac{u + \omega_s c^2}{2\sigma^2} \right) I_0 \left( \frac{\omega_s c \sqrt{u}}{2\sigma^2} \right),$$

$$p_V(v|H_0) = \frac{1}{2\sigma^2} \exp \left( -\frac{v}{2\sigma^2} \right),$$

$$p_V(v|H_1, A) = \frac{1}{2\sigma^2} \exp \left( -\frac{v + (\omega_m a^2)}{2\sigma^2} \right) I_0 \left( \frac{\omega_m a \sqrt{v}}{\sigma^2} \right),$$

$$p_V(v|H_2, C) = \frac{1}{2\sigma^2} \exp \left( -\frac{v + (\omega_s c^2)}{2\sigma^2} \right) I_0 \left( \frac{\omega_s c \sqrt{v}}{\sigma^2} \right),$$

where noise variance in the main and auxiliary channels, $\sigma_m^2$ and $\sigma_s^2$, are taken as $2\sigma^2$ and $I_0$ denotes the modified Bessel function of the first kind. Considering the SLB structure given in Figure 2, $P_B$ as a function of $C$ (amplitude of the interference signal in sidelobe) can be written as

$$P_B(c) = \text{Prob} \left\{ \frac{V}{U} > F|H_2, C \right\} = \int_0^\infty \int_0^{v/F} p_{U,V}(u,v|H_2,C)dudv,$$

$$= \int_0^{v/F} \int_0^\infty p_U(u|H_2, C) p_V(v|H_2, C)dudv. \tag{5}$$

In the last line of (5) independence of $U$ and $V$ is used since the noise in the main and the auxiliary channels, namely $W_r$ and $W_s$, are independent. (5) indicates that $P_B(c)$ depends on the joint probability distribution function of $U$ and $V$, $p_{U,V}(u,v)$, conditioned on $H_2$ hypothesis. Following to the results given in [5], $P_B$ is derived as a function of $C$. Then, pdf of $C$ is employed and $P_B(c)$ is averaged with respect to $C$ to derive $P_B$ as follows

$$P_B = \int_0^\infty P_B(c)p_C(c)dc. \tag{6}$$

The resultant expression for $P_B$ can be given as

$$P_B = \frac{1}{2} \left[ 1 - \frac{\text{INR}(F - (\omega_s/\delta)^2) - (F + 1)}{\text{SNR}(F - (\omega_m^2) + (F + 1))} \right].$$

Note that interference to noise ratio of the interfering source in the main channel is taken as

$$\text{INR} = \frac{E\{\sigma^2\}}{2\sigma^2} = \frac{\sigma_C^2}{2\sigma^2}, \tag{8}$$

where $\sigma_C^2$ is the average power of the fluctuating interference in the sidelobe as shown in (2).

To evaluate the performance of the SLB structure, $P_{TB}$ is also to be derived. Considering the SLB structure, $P_{TB}$ as a function of $A$ (amplitude of the target in the mainlobe) can be written as

$$P_{TB}(a) = \text{Prob} \left\{ \frac{V}{U} > F|H_1, A \right\} = \int_0^\infty \int_0^{v/F} p_{U,V}(u,v|H_1,A)dudv. \tag{9}$$

Following the same steps with the derivation of $P_B$, $P_{TB}$ can be written as follows

$$P_{TB} = \int_0^\infty P_{TB}(a)p_A(a)da. \tag{10}$$

and a closed form expression for $P_{TB}$ can be found as,

$$P_{TB} = \frac{1}{2} \left[ 1 - \frac{\text{INR}(F - (\omega_s/\delta)^2) - (F + 1)}{\text{SNR}(F - (\omega_m^2) + (F + 1))} \right].$$

SNR denotes the signal to noise ratio and it can be written as

$$\text{SNR} = \frac{E\{\sigma^2\}}{2\sigma^2} = \frac{\sigma_A^2}{2\sigma^2}, \tag{12}$$

where $\sigma_A^2$ is the average power of the fluctuating target in the mainlobe.

IV. NUMERICAL RESULTS

The variation of two probabilities, $P_B$ and $P_{TB}$, in different scenarios is presented in order to illustrate the advantage of the complementary pattern over the conventional auxiliary antenna pattern.

To examine the efficiency of the proposed radiation pattern, the parameter $x$ (shown in Figure 3), is introduced. This parameter denotes the ratio between the side lobe and the main lobe level of the auxiliary pattern.

$$x = \frac{\omega_s^2}{\omega_m^2}. \tag{13}$$

When $x = 1$, the results correspond to the conventional SLB performance, and the performance of the proposed SLB system is studied for the cases of $x > 1$. 

978-1-5386-4167-5/18/$31.00 ©2018 IEEE 1180
During the analysis, main antenna gain is kept at 0 dB. The effect of $x$ on the system performance is investigated for various SNR, INR, $\delta^2$, $\omega^2$, and $F$.

The first example demonstrates the probability of blanking, $P_B$, as a function of the parameter $x$, by setting the probability of target blanking, $P_{TB}$, at a desired value. The Swerling-1 target model is assumed. More specifically, in Figure 4, the probability of target blanking, $P_{TB}$, is kept constant and INR is varied and the probability of blanking, $P_B$, is plotted as a function of $x$. Table I demonstrates the system parameters used in the numerical experiment.

Table I. System Parameters

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<th>Value</th>
<th>Explanation</th>
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<td>$-20$ dB</td>
<td>$\delta^2$, main pattern sidelobe level</td>
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<tr>
<td>$-10$ dB</td>
<td>$\omega^2$, auxiliary pattern level</td>
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</tbody>
</table>

According to the expression in (11), 20 dB SNR and the system parameters given in Table I results in $P_{TB} = 0.0011$. Keeping the value of $P_{TB}$ constant, increasing the parameter $x$ over 0 dB causes a decrease in $F$ and thereby an increase in $P_B$. To clarify, each increment in $x$ would set a new threshold which allows to achieve better $P_B$ values. In Figure 4, probability of blanking, $P_B$, is plotted as a function of $x$ for various INR. Left y-axis denotes the probability of blanking, $P_B$, and the right y-axis indicates the threshold values for the same x-axis.

Figure 4 indicates that the larger the parameter $x$ is, the higher the probability of blanking is. In fact, 20 dB increment on $x$ provides a drastic improvement for the blanking performance against the interferers having high INRs. It is expected that the probability of blanking increases as INR gets larger, since the presence of an interference is easily discernible with increasing INR in a noisy channel. Another result that can be drawn from Figure 4 is that improving the value of $x$ over 20 dB does not change the performance significantly.

It is important to note that threshold, $F$, cannot be larger than $\frac{20}{\sigma^2}$ for the proper operation of Maisel type SLB systems [2]. Here, considering that fact, $F$ is set to its maximum value, 10 dB, for $x = 0$ dB. In order to investigate the effect of the threshold, $F$, on the proposed system, the probability of blanking is plotted in Figure 5 by setting the threshold, $F$, to 5 dB (when $x = 0$ dB). It has to be underlined that decreasing the threshold value leads to an increase in $P_{TB}$ from 0.0011 to 0.0037.

![Fig. 5. Probability of blanking, $P_B$, and threshold, $F$, vs parameter $x$ for different INR values](image)

In Figure 5, the blanking probability improvement is not as significant as in Figure 4. Thus, it can be inferred that the performance improvement due to the proposed SLB structure is the most significant when the threshold, $F$, is selected close to its maximum.

Another way to investigate the effect of $x$, i.e. the dip of the auxiliary pattern, on the system performance is to plot the probability of target blanking, $P_{TB}$, as a function of SNR for different values of $x$. In Figure 6, for the same SNR values (x-axis), left y-axis denotes the probability of target blanking, $P_{TB}$, and the right y-axis denotes the performance gap between conventional and complementary auxiliary patterns. Performance gap can be defined as the ratio between the SNRs of the conventional and the complementary auxiliary channels for the same probability of target blanking, $P_{TB}$, values. In Figure 6, the threshold, $F$, is taken as 0 dB and the system parameters are taken as given in Table I.

According to the Figure 6, $P_{TB}$ performance of a conventional system for a 20 dB target can be provided with a 18.7 dB target with proposed system. It is worthy of note that even if the performance gap improves with increasing SNR, the effect of the performance gap on $P_{TB}$ becomes less significant.

Table II. System Parameters

<table>
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<tr>
<th>Value</th>
<th>Explanation</th>
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<tbody>
<tr>
<td>$-30$ dB</td>
<td>$\delta^2$, main pattern sidelobe level</td>
</tr>
<tr>
<td>$-20$ dB</td>
<td>$\omega^2$, auxiliary pattern level</td>
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</table>

The overall system performance of a second system with high main-to-sidelobe gain antenna is demonstrated in Figures 7 and 8. System parameters are given in Table II.
Fig. 6. Probability of target blanking, $P_{TB}$, and performance gap vs SNR for different $x$ values, $F = 0$ dB

Fig. 7. Probability of blanking, $P_B$, and threshold, $F$, vs parameter $x$ for different INR values

V. CONCLUSIONS

In this paper, a complementary auxiliary antenna pattern configuration is proposed to reduce the coupling between the probability of blanking and the probability of target blanking in the conventional Maisel’s structure. The well known expressions for $P_{TB}$ and $P_B$ are modified according to the complementary pattern, for Swerling 1 target model. It is found that the complementary auxiliary pattern is advantageous over the conventional pattern in terms of interferer blanking and the target blanking performance. In particular, rather drastic improvements are observed for antennas having low main-to-sidelobe ratios. When the threshold, $F$, is set to its maximum, it is demonstrated that the blanking probability, $P_B$, of an interferer, having 20 dB INR, can be increased from 0.48 to 0.72. It is also shown that for a fixed blanking probability blanking of a 20 dB mainlobe target occurs at 1.3 dB smaller SNR with the complementary pattern. On the other hand, it is important to note that the contribution of the complementary antenna might becomes quite negligible for main antennas having sufficiently good main-to-sidelobe ratios.

REFERENCES