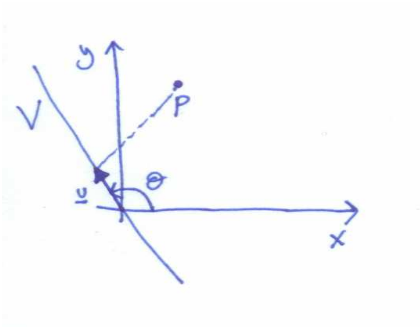
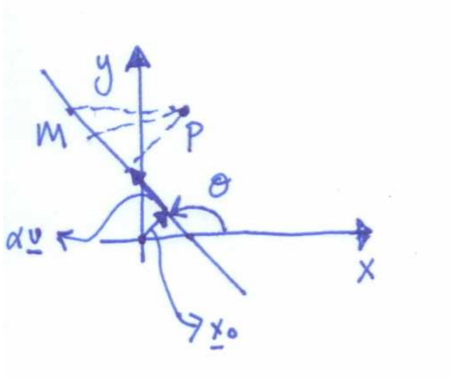
EE 503 - HW #1
(Due: Oct. 21, 2015)



1. P is a point in the x-y plane. V is 1-dimensional sub-space of (x,y) plane.

   $$V = \{(x,y) : (x,y) = \alpha\,(\cos\Theta, \sin\Theta),\ \alpha \in R\}$$

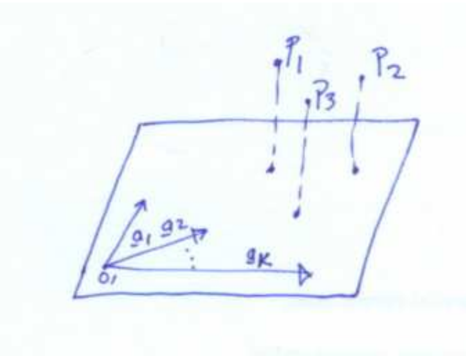
   i. Find the point in V which is closest to P in the Euclidean sense.
      a. By using orthogonality of the projection error to the sub-space
      b. By optimization over α.



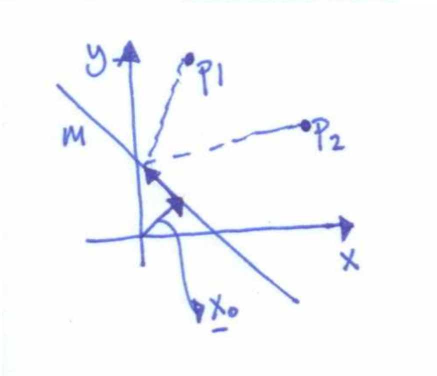
2. P is a point in x-y plane. M is 1-dimensional linear variety of (x,y) plane (2-dimensional space).

$$M = \{(x,y) : (x,y) = \underline{x}_0 + \alpha\,(\cos\Theta, \sin\Theta),\ \alpha \in R\}$$

   i. Show that a linear variety is not a vector-space. (You may research on linear variety from internet)
   ii. Find the point in M which is closest to P (in the Euclidean sense), by optimizing over α.
   iii. Comment on the result found in part-ii. Is the orthogonality principle valid for linear variety ?



3. P_1, P_2 and P_3 are points in N dimensional space. Let S be the sub-space spanned by $\{a_1, a_2, \ldots, a_K\}$.
   i. Find the point $\hat{P}$ in S such that $\|\hat{P} - P_1\|^2 + \|\hat{P} - P_2\|^2 + \|\hat{P} - P_3\|^2$ is minimum. ($\|x\|$ is the Euclidean norm.)
   ii. Give a geometric interpretation.



4. P_1 and P_2 are two points in the x-y plane. M is 1 dimensional linear variety of (x,y) plane (2-dimensional space).

$$M = \{(x,y) : (x,y) = \underline{x}_0 + \alpha\,(\cos\Theta, \sin\Theta),\ \alpha \in R\}$$

Let P_1=(1,0), P_2=(-1,0) and let M be the points on the line $y = -x + 4$. Find the point $\hat{P}$ in the variety M such that the sum of distances to P_1 and P_2, i.e. $\|\hat{P} - P_1\| + \|\hat{P} - P_2\|$, is minimum. (Note: This problem is different from the previous one. Here the cost is the distance itself, not the sum of distance *squares*).
Hint: Consider drawing ellipses with the foci points P_1 and P_2. (You may check http://torus.math.uiuc.edu/eggmath/Shape/ellipse-eq.html for more information.)