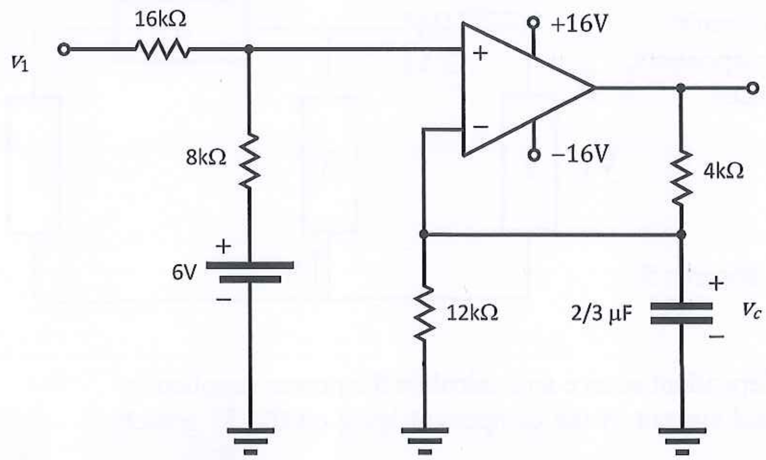
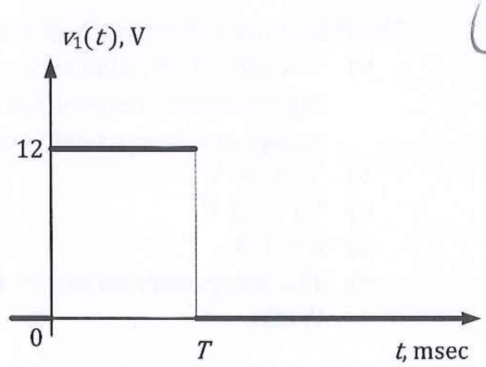
Question 5 (20 pts)

(1)





$T = 4\ln(2)$ msec

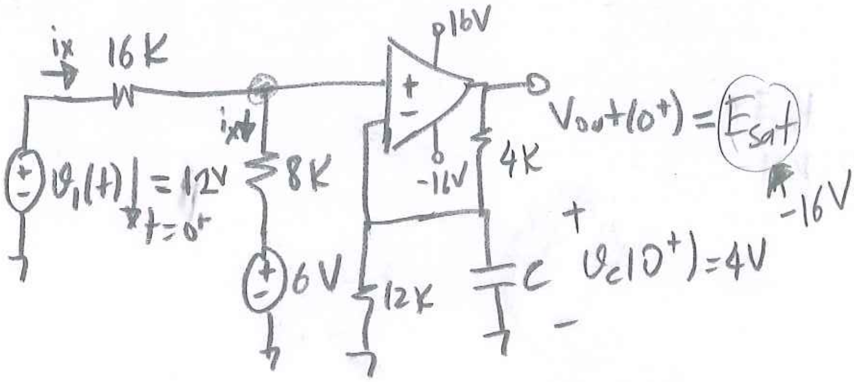
$v_c(0^-) = 4\,V.$

Find and sketch $v_c(t)$ for $t \geq 0$.

Soln.

We need to find the state of op-amp (its operating region) at $t=0^+$ to start the solution. We know that $v_c(0^+) = v_c(0^-) = 4$ V.

$\boxed{\text{Assume } -SAT \text{ at } t=0^+:}$



$v_+ = ?$ $\quad i_x = \frac{12-6}{24}\,mA = \frac{1}{4}\,mA.$

$v_+ = 12 - (16K)\,i_x = 8$ Volts.

$v_d = v_+ - v_- = 8 - 4 = 4 > 0$

but for the assumption to be valid $v_d < 0$! ; so assumption is not collect.

$\boxed{\text{Assume linear Region for op-amp at } t=0^+}$ → $|v_{out}| < 16V$
→ $v_d = 0$

but $v_+ = 8V$ and $v_- = 4V$ → So, can not be linear region either.

Assume +SAT: (This should be the answer! (why?))

$V_{out} = 16V = +E_{sat}$ }
$V_d > 0$

$V_d = V_+ - V_- = 4V > 0$
$V_+ = 8V$, $V_- = 4V$

Indeed, this is the correct region.

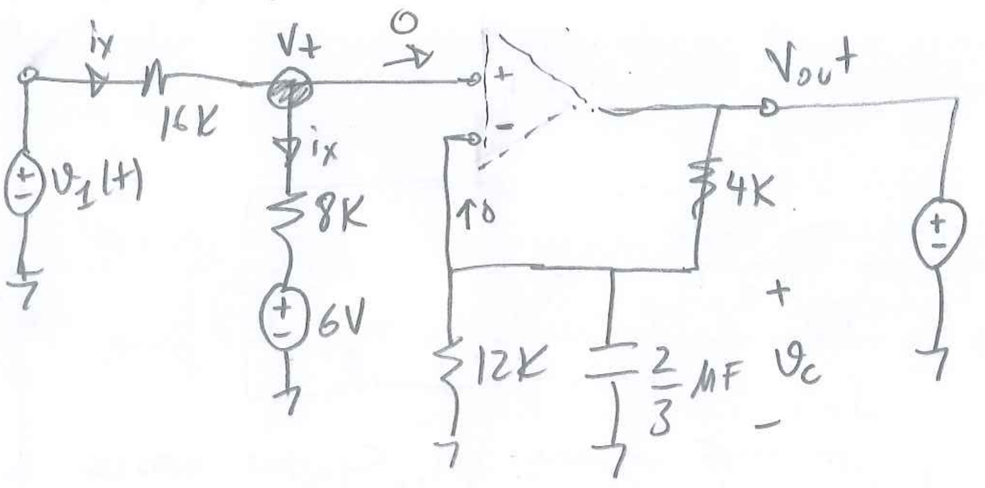
Note: Op-amp operates under negative feedback, hence at a given capacitor voltage, there is one and only one operation region for op-amp. If op-amp has a positive feedback connection also and positive feedback dominates negative feedback, then there can be a case (cap. voltage) that op-amp can be in all 3 regions. For such a problem, additional information should be provided to determine the $t=0^+$ operational region.

In this case, we are sure that op-amp is in +SAT region at $t=0^+$.

$, 0 < t < T_x \leftarrow ?$

③

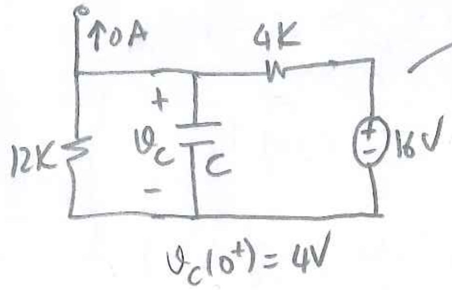
$\boxed{v_c(0^+) = 4V}$



Op-amp output at +SAT

(1) $V_+ = 6 + \underbrace{\left(\frac{V_1 - 6}{24}\right)}_{i_x} \cdot 8 = \frac{V_1}{3} + 4$ Volts,

$V_- = V_c = ? \longrightarrow$

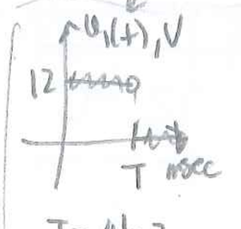


First order circuit

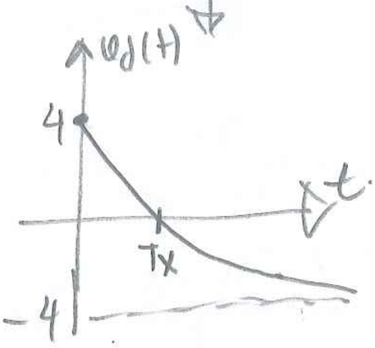
$\boxed{v_c(t) = 12 - 8e^{-t/\tau} \text{ Volts}}$

$\boxed{\tau = (12 // 4)\,\frac{2}{3} \text{ msec} = 2 \text{ msec}}$

$v_d = V_+ - V_- = \frac{V_1}{3} - 8 + 8e^{-t/\tau} \Rightarrow v_d(t) = -4 + 8e^{-t/\tau} \quad 0 < t < T_x < T$



$T = 4\ln 2$

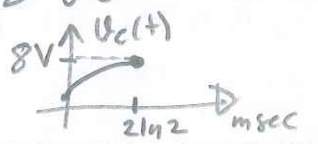


$\Rightarrow$ Note that when $t > T_x$

Op-amp leaves +SAT since $v_d > 0$ not valid.

$v_d(T_x) = 0 = -4 + 8e^{-T_x/\tau} \Rightarrow T_x = \tau \ln 2 = 2\ln 2$ msec

Then, $v_c(t) = 12 - 8e^{-t/\tau}$ is valid for $0 < t < T_x = 2\ln 2$ msec.

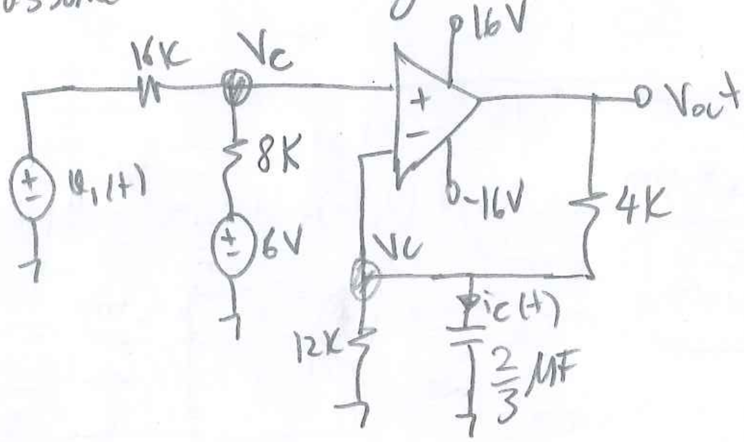


what's the state of op-amp at $t = T_x^+ = 2\ln 2^+$ msec.? (4)

$V_+(T_x^+) = 8$ Volts.
$V_-(T_x^+) = 8$ Volts $\rightarrow \vartheta_d(T_x^+) = 0V \rightarrow$ seems to be at linear region.

Let's assume linear region and check:

$\boxed{T_x^+ < t < T}$



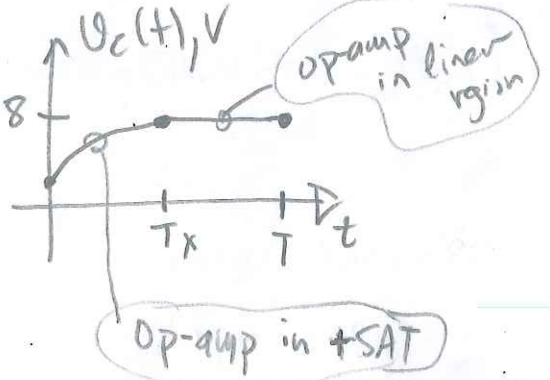
$\vartheta_c(T_x^+) = 8$ Volts.

$\vartheta_+ = \vartheta_c(t) = 8$ Volts
↑ constant!

$T_x^+ < t < T \rightarrow i_c(t) = 0 \rightarrow V_{out} = \frac{8}{12} \cdot 16 = \frac{32}{3}$ Volts

$\downarrow$

Opamp is indeed in linear region $\leftarrow |\vartheta_{out}| < E_{sat} = 16V$



what is the state of op-amp at $t = T^+$?

$V_+(T^+) = \frac{V_1(T^+)^{\,0}}{3} + 4 = 4V$ (1) page 3
$V_-(T^+) = V_c(T^+) = 8V$

$\vartheta_d = 4 - 8 = -4 < 0$

$\downarrow$

Op-amp seems to enter -SAT

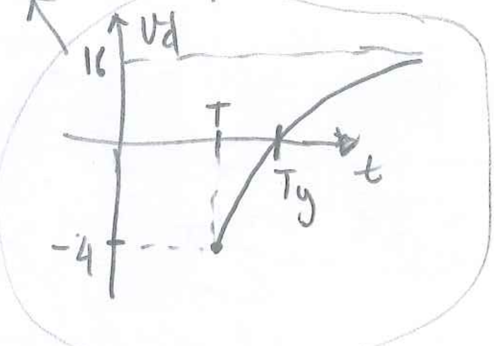
Let's assume $-SAT$ then for $t > T$.

As in page 3, with some minor changes

$$v_c(t) = -12 + 20e^{-(t-T)/\tau} \text{ Volts}, \quad t > T$$

$\tau = 2\text{msec}$

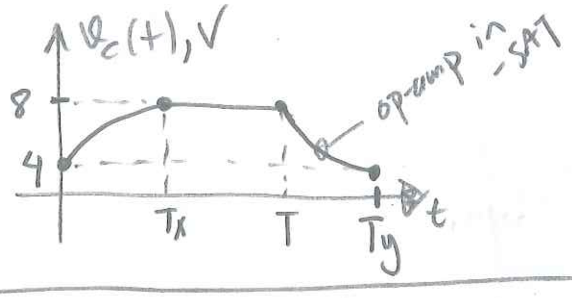
$$v_d(t) = \overset{4}{v_+(t)} - v_c(t)$$
$$= 16 - 20e^{-(t-T)/\tau}, \quad t > T$$



$-SAT$ assumption is valid between $T < t < T_y$ since $v_d < 0$ is not satisfied $t > T_y$! $v_d(T_y) = 0 \rightarrow T_y = T + \tau \ln\frac{5}{4}$

$T = 4\ln 2$ msec

$$\boxed{T_y = 2\ln 5 \text{ msec}}$$



What is op-amp state at $t = T_y^+$?

Assume linear,

Check results given page 4 and

$v_c(t) = 4$ V, $t > T_y \rightarrow i_c(t) = 0$, $t > T_y \rightarrow V_{out} = \frac{4}{12} \cdot 16 < 16$

Op-amp is indeed in linear

Final Answer →