P.1: (Hayes Prob. 3.9) Determine whether or not the following are valid autocorrelation matrices:

\[ R_1 = \begin{bmatrix} 4 & 1 & 1 \\ -1 & 4 & 1 \\ -1 & -1 & 4 \end{bmatrix}; \quad R_2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}; \quad R_3 = \begin{bmatrix} 1 & 1 + j \\ 1 - j & 1 \end{bmatrix} \]

\[ R_4 = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}; \quad R_5 = \begin{bmatrix} 2 & j & 1 \\ -j & 4j & -j \\ 1 & j & 2 \end{bmatrix} \]

P.2: (Hayes Prob. 3.10) The input to a LTI filter with impulse response

\[ h(n) = \delta(n) + \frac{1}{2}\delta(n - 1) + \frac{1}{4}\delta(n - 2) \]

is zero mean wide-sense stationary process with autocorrelation \( r_x(k) = \left(\frac{1}{2}\right)^{|k|} \).

a) What is the variance of the output process?
b) Find the autocorrelation of the output process, \( r_y(k) \).

P.3: (Hayes Prob. 3.13) Suppose we are given a zero mean process \( x(n) \) with autocorrelation

\[ r_x(k) = 10 \left(\frac{1}{2}\right)^{|k|} + 3 \left(\frac{1}{2}\right)^{|k-1|} + 3 \left(\frac{1}{2}\right)^{|k+1|} \]

a) Find a filter which, when driven by unit variance white noise, will yield a random process with this autocorrelation.
b) Find a stable and causal filter which, when excited by \( x(n) \), will produce zero mean, unit variance white noise.

P.4: (Haykin Prob. 2.21) Consider an autoregressive process \( u(n) \) of order 2, described with the following relation: \( u(n) = u(n - 1) - 0.5u(n - 2) + v(n) \), where \( v(n) \) is a white noise of zero mean and variance 0.5.

a) Write Yule-Walker equations for the process.
b) Solve these two equations for the autocorrelation functions value \( r_u(1) \) and \( r_u(2) \).
c) Find the variance of \( u(n) \).