**MATH 535, Topology, Homework 1**

**1. (2+3+3 pts)**
Let $G$ be a group. Our aim in this question is to first endow $\mathcal{P}(G)$ with a topology and then construct *the space of subgroups of* $G$ as a subspace of $\mathcal{P}(G)$.

For each **finite** subset $F, K \subseteq G$, let $U_{F,K} = \{S \in \mathcal{P}(G) : \ F \subseteq S, \ K \subseteq S^c\}$.

a) Show that the collection

$$\mathcal{B} = \{U_{F,K} \subseteq \mathcal{P}(G) : \ F, K \text{ are finite subsets of } G\}$$

is a basis for a topology on $\mathcal{P}(G)$.

For the remainder of this question, you will consider $\mathcal{P}(G)$ with the topology $\tau_{\mathcal{B}}$ where $\tau_{\mathcal{B}}$ is the topology generated by the basis $\mathcal{B}$ in part a.

b) Show that the set

$$\operatorname{Sub}(G) = \{H \in \mathcal{P}(G) : \ H \text{ is a subgroup of } G\}$$

is a closed subset of $\mathcal{P}(G)$.

> **Hint**. Show that the complement of this set is open. In order to do that, ask yourself this: When is a subset $H \subseteq G$ not a subgroup? Is the set of $H$'s which do not contain the identity an open set? Is the set of $H$'s which do not contain the inverse of some element an open set? Is the set of $H$'s which in not closed under group multiplication an open set?

For the last part of this question, suppose that $G = \mathbb{Z}$ and consider $\operatorname{Sub}(\mathbb{Z})$ with the subspace topology as a subspace of $\mathcal{P}(\mathbb{Z})$.

c) Show that $\lim\limits_{n\to\infty} n\mathbb{Z} = \{0\}$ in the topological space $\operatorname{Sub}(\mathbb{Z})$.

**2. (2 pts)** Let $X$ be a topological space and $A \subseteq X$. Prove that if $A$ is closed, then $A'$ is closed.

**3. (2 pts)** Let $(X, \preceq)$ be a linearly ordered set. Prove that the corresponding order topology on $X$ is Hausdorff.