************* PLEASE WRITE YOUR NAME CLEARLY USING CAPITAL LETTERS *************		
F U L L N A M E	S T U D E N T I D	Deadline: February 3, 2021, 12:30 5 questions on 4 pages 100 points in total

1. (4 × 5 pts) Determine whether the following statements are true or false. If the statement is true, then provide a **brief** proof of the statement. If the statement is false, then provide a counterexample and **briefly** explain why it is a counterexample.

a) **True or False:** If X is a non-metrizable compact Hausdorff space, then X is not second-countable.

b) **True or False:** If X is a non-compact topological space, then any image of X under a continuous injection is non-compact.

c) **True or False:** If X is a discrete space which is also sequentially compact, then X is finite.

d) **True or False:** If X is a topological space and $Y \subseteq X$ is meager in the topological space X, then every $Z \subseteq Y$ is meager **in the subspace** Y**.**

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2. (10+10+10 pts) Consider the Cantor space $2^{\mathbb{N}}$ with its usual product topology generated by the basis

$$\mathcal{B} = \{U_{F,K} \subseteq 2^{\mathbb{N}} : \ F, K \text{ are finite subsets of } \mathbb{N}\}$$

where

$$U_{F,K} = \{(a_i)_{i\in\mathbb{N}} : \forall i \in F \ a_i = 1 \ \text{ and } \forall i \in K \ a_i = 0 \ \}$$

a) Show that the metric given by

$$d((a_i)_{i\in\mathbb{N}}, (b_i)_{i\in\mathbb{N}}) = \begin{cases} 0 & \text{if } \ (a_i)_{i\in\mathbb{N}} = (b_i)_{i\in\mathbb{N}} \\ \dfrac{1}{2^{i+1}} & \text{if } \ (a_i)_{i\in\mathbb{N}} \neq (b_i)_{i\in\mathbb{N}} \text{ and } i = \min\{k \in \mathbb{N} : a_k \neq b_k\} \end{cases}$$

is compatible with the topology of $2^{\mathbb{N}}$.

b) Show that every element of $\mathcal{B}$ is clopen.

c) Show that any clopen subset of $2^{\mathbb{N}}$ is a finite union of elements of $\mathcal{B}$.
(**Hint.** Recall that $2^{\mathbb{N}}$ is a compact space. Take any clopen subset C of $2^{\mathbb{N}}$. Can you cover it with appropriate open sets from $\mathcal{B}$?)

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3. (10+10 pts) Consider the cartesian product $\mathcal{A} = [0,1] \times \{0,1\}$ endowed with the order topology induced by the lexicographic order defined as

$$(x, y) \preceq (x', y') \text{ if and only if } x \prec x', \text{ or, } x = x' \text{ and } y \preceq y'$$

a) Show that $\mathcal{A}$ is compact Hausdorff.

b) Show that the subspace $\mathcal{A}_1 = (0,1) \times \{1\}$ of $\mathcal{A}$ is homeomorphic to the subspace $(0,1) \subseteq \mathbb{R}_\ell$. Recall that $\mathbb{R}_\ell$ is the Sorgenfrey line whose topology is generated by the sets of the form $[a, b)$ with $a < b$.

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4. (10+10 pts) Consider the product space $2^{\mathbb{R}}$ where each component $2 = \{0, 1\}$ has the discrete topology.

a) Show that $2^{\mathbb{R}}$ is not first-countable.

b) Show that there is no continuous surjection $f : 2^{\mathbb{R}} \to 2^{\mathbb{R}}$ where the domain is endowed with the product topology and the codomain is endowed with the box topology.

5. (10 pts) Let G be a topological group and $H \leqslant G$ be a subgroup. Endow G/H with the (unique quotient) topology which makes the map $\pi : G \to G/H$ given by $\pi(x) = xH$ a quotient map. Show that π is an open map.