PLEASE WRITE YOUR NAME CLEARLY USING CAPITAL LETTERS		
F U L L N A M E	S T U D E N T I D	DURATION 100 MINUTES
4 QUESTIONS ON 4 PAGES		TOTAL 50 POINTS

By signing below, I pledge that I will write this examination as my own work and without the assistance of others or the usage of unauthorized material or information. I understand that possession of any kind of electronic device during the exam is prohibited. I also understand that not obeying the rules of the examination will result in immediate cancellation and disciplinary procedures.

Signature ....

***(12 pts)* 1.** Let $F(x) = \lfloor x \rfloor$ be the floor function. Consider the Lebesgue-Stieltjes measure $\mu_F : \mathcal{B}(\mathbb{R}) \to [0, \infty]$ associated to F. Show that $\mathbb{R} - \mathbb{Z}$ is μ_F-null.

(12 pts) 2. Compute $\lim_{n\to\infty} \int_{(0,\infty)} \frac{n\sin(x/n)}{x(1+x/n)^n}\, d\mathbf{m}$. Explain each step in detail by referring to the relevant theorems.

(13 pts) **3.** Let $(X, \mathcal{M}, \mu)$ be a measure space and fix a map $f \in L^+(X, \mathcal{M}, \mu)$. Consider the map $\eta : \mathcal{M} \to [0, +\infty]$ given by

$$\eta(E) = \int_E f \, d\mu$$

You are **given** that η is a measure. Prove that, for every $g \in L^+(X, \mathcal{M}, \eta)$, we have

$$\int_X g \, d\eta = \int_X fg \, d\mu$$

(13 pts) **4.** Let $(\mathbf{X}, \mathcal{M}, \mu)$ be a measure space with $0 < \mu(\mathbf{X}) < +\infty$. Consider the function $\rho : L(\mathbf{X}, \mathcal{M}, \mu) \times L(\mathbf{X}, \mathcal{M}, \mu) \to [0, +\infty)$ given by

$$\rho(f, g) = \int_X \min\{|f - g|, 1\} \, d\mu$$

Let $f \in L(\mathbf{X}, \mathcal{M}, \mu)$ and let $(f_n)_{n \in \mathbb{N}}$ be a sequence of functions in $L(\mathbf{X}, \mathcal{M}, \mu)$. Show that if $f_n \to f$ in measure, then for all $\epsilon \in \mathbb{R}^+$ there exists $k \in \mathbb{N}$ such that for all $n \in \mathbb{N}$ with $n \geq k$, we have that $\rho(f_n, f) < \epsilon$.