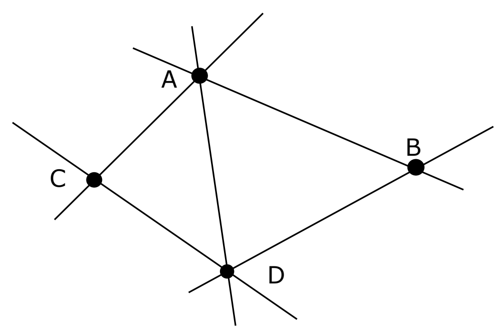
M E T U Department of Mathematics

Math 111 Fundamentals of Mathematics Fall 2019 Midterm I 17 October 2019 17:40		
F U L L N A M E	S T U D E N T I D	DURATION 70 MINUTES
5 QUESTIONS ON 2 PAGES		TOTAL 40 POINTS

By signing below, I pledge that I will write this examination as my own work and without the assistance of others or the usage of unauthorized material or information. I understand that possession of any kind of electronic device during the exam is prohibited. I also understand that not obeying the rules of the examination will result in immediate cancellation and disciplinary procedures.

Signature ....

(3+3+3+3 pts) **1.** Consider the points A, B, C, D in the plane and the lines connecting these points:



Let $L(x,y)$ be the expression

"x and y lie on the same line"

where the possible values of x and y are from the set $\{A, B, C, D\}$.

Determine whether the following statements are true or false. In each case, briefly explain your reasoning.

a) $\exists x\ \forall y\ L(x,y)$ This statement is TRUE. Because if we choose $x = A$, then we see that $L(x,y)$ is true for any y.

b) $\exists x\ \exists y\ (\neg L(x,y))$ This statement is TRUE. Because if we choose $x = B$ and $y = C$, then we see that $L(x,y)$ is false.

c) $\forall x\ \forall y\ L(x,y)$ This statement if FALSE. Because, for $x = B$ and $y = C$, we have that $L(x,y)$ is false. Thus, it is not the case that $L(x,y)$ holds for every x and y. (Alternative solution: This statement is equivalent to the negation of the statement in part b) which was true. Therefore it is FALSE.)

d) $\exists x\ \exists y\ \exists z\ \Big[L(x,y) \wedge L(y,z) \wedge \neg L(x,z)\Big]$

This statement is TRUE. Because, if we choose $x = C, y = A, z = B$, then we have that $L(x,y)$ and $L(y,z)$ are true and $L(x,z)$ is false.

(4+4 pts) **2.** Let P be the statement $\exists x\ \Big[\ \Big(\forall y\ \ G(x,y)\Big)\ \wedge\ \Big(\exists z\ \ (E(z) \to G(x,z))\Big)\ \Big]$ where the variables x and y range over the positive integers.

a) Find the negation of P.

$\forall x\ \Big[\ \Big(\exists y\ \ \neg G(x,y)\Big)\ \vee\ \Big(\forall z\ \ (E(z) \wedge \neg G(x,z))\Big)\ \Big]$

b) Let $G(x,y)$ be the expression "$x = y$" and $E(z)$ be the expression "z is even". Determine whether P is true or false.

In this case P is FALSE. In order to see this, we shall show that the negation of P is true. Let x be arbitrary. Choose $y = x + 1$. Then $G(x,y)$ is false, and hence $\Big(\exists y\ \ \neg G(x,y)\Big)$ is true. Thus, $\forall x\ \Big[\ \Big(\exists y\ \ \neg G(x,y)\Big)\ \vee\ \Big(\forall z\ \ (E(z) \wedge \neg G(x,z))\Big)\ \Big]$ is true.

(4+2 pts) **3.** Consider the following argument:

It is not sunny today and it is colder than yesterday. If we go swimming, then it will be sunny today. If we do not go swimming, then we will not take a canoe trip. Therefore, if we take a canoe trip then we will go home by the sunset.

- **S**: It is sunny today
- **C**: It is colder than yesterday
- **G** : We will go swimming
- **K**: We will take a canoe trip
- **H**: We will be home by sunset

a) Express this argument using the statements on the left and logical connectives.

$\neg S \wedge C$
$G \to S$
$(\neg G) \to \neg K$

$K \to H$

b) Determine whether the set of premises of this argument is consistent or not.

If we let S be FALSE, C be TRUE, G be FALSE and K be FALSE, then all premises become true. Hence, this set of premises is consistent.

(7 pts) **4.** Write a derivation for the following valid argument.

$P \to Q$
$Q \to (S \vee \neg P)$
$(\neg R) \wedge P$

S

1) $P \to Q$
2) $Q \to (S \vee \neg P)$
3) $(\neg R) \wedge P$
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4) P from 3) and simplification $(A \wedge B \implies B)$
5) Q from $1), 4)$ and Modus Ponens $((A \to B) \wedge A \implies B)$
6) $S \vee \neg P$ from $2), 5)$ and Modus Ponens $((A \to B) \wedge A \implies B)$
7) $P \to S$ from 2) and $((A \to B) \iff (B \vee \neg A))$
8) S from $4), 7)$ and Modus Ponens $((A \to B) \wedge A \implies B)$

(7 pts) **5.** Write a derivation for the following valid argument.

$\exists x \; \neg Q(x)$
$\forall x \; \big((\neg P(x)) \to Q(x)\big)$
$\forall x \; (P(x) \to \neg R(x))$

$\neg(\forall x \; R(x))$

1) $\exists x \; \neg Q(x)$
2) $\forall x \; \big((\neg P(x)) \to Q(x)\big)$
3) $\forall x \; (P(x) \to \neg R(x))$
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4) $\neg Q(a)$ from 1) and Existential Instantiation
5) $(\neg P(a)) \to Q(a)$ from 2) and Universal Instantiation
6) $\neg\neg P(a)$ from $4), 5)$ and Modus Tollens $(A \to B) \wedge \neg B \implies \neg A$
7) $P(a)$ from 6) and double negation $\neg\neg A \Leftrightarrow A$
8) $P(a) \to \neg R(a)$ from 3) and Universal Instantiation
9) $\neg R(a)$ from $7), 8)$ and Modus Ponens $(A \to B) \wedge A \implies B$
10) $\exists x(\neg R(x))$ from 9) and Existential Generalization
11) $\neg(\forall x \; R(x))$ from 10) and de Morgan's law $\exists x(\neg R(x)) \iff \neg(\forall x \; R(x))$