MATH 501, Analysis, Homework 7

1. (2 pts) Let μ and ν be signed measures on the measurable space $(\mathbf{X}, \mathcal{M})$. Prove that $\mu \perp \nu$ if and only if $\mu \perp \nu^+$ and $\mu \perp \nu^-$.

2. (3 pts) Let $(\mathbf{X}, \mathcal{M}, \mu)$ be a measure space and let $f : \mathbf{X} \to \mathbb{R}$ be in $L^1(\mathbf{X}, \mathcal{M}, \mu)$. Consider the measure defined by $\nu(E) = \int_E f \, d\mu$ for all $E \in \mathcal{M}$. Find the Hahn decomposition of the signed measure space $(\mathbf{X}, \mathcal{M}, \nu)$ and the Jordan decomposition of ν in terms of f and μ.

3. (3 pts) Let $\mu : \mathcal{P}(\mathbb{N}) \to [0, \infty]$ and $\nu : \mathcal{P}(\mathbb{N}) \to [0, \infty]$ be the measures on the measurable space $(\mathbb{N}, \mathcal{P}(\mathbb{N}))$ given by

$$\mu(A) = \sum_{n \in A} n^2$$

and

$$\nu(A) = \sum_{n \in A} 2n$$

Show that $\mu \ll \nu$ and find the Radon-Nikodym derivative $\dfrac{d\mu}{d\nu}$.

4. (2 pts) Consider the measures $\mathbf{m}$ and μ on the measurable space $([0,1], \mathcal{B}([0,1]))$, where $\mathbf{m}$ is the Lebesgue measure and μ is the counting measure. Prove that $\mathbf{m} \ll \mu$ and there does not exists a measurable function $f : [0,1] \to [0, \infty)$ such that $\mathbf{m}(A) = \int_A f \, d\mu$.

Solutions