

# **Meaning, Form and Adjacency: Schönfinkel's Legacy**

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Moses Ilyich Schönfinkel



1889 Dnipropetrovsk, Ukraine – 1942 Moscow

December 7, 1920, Göttingen seminar to David Hilbert's research group

$$f(x) = x + 1$$

$$f(3) = 3 + 1$$

$$\lambda x.x + 1$$

$$\lambda x.x + 1(3) =_{\beta} 3 + 1$$

$$f(x, y) = x - y$$

$$f(3, 5) = 3 - 5$$

$$\lambda x \lambda y.x - y$$

$$\lambda x \lambda y.x - y(3) =_{\beta} \lambda y.3 - y$$

$$\lambda y \lambda x.x - y$$

$$\lambda y \lambda x.x - y(5) =_{\beta} \lambda x.x - 5$$

Two definitions of  $f(x, y)$  must be related

Polish notation. Functions first:  $f(x, y) = x - y = -xy$

$-xy = -yx$  is clearly wrong

$-xy = \mathbf{C} - yx$  is fine

if  $\mathbf{C}fbc = fcb$

$\mathbf{C}$  is Schönfinkel's **interchange**,  $\mathbf{T}$  (nowadays called **permutator**)

Relating functions in function-first notation:  $f(gx)$

$f(gx)$  depends on  $x$  because  $g$  depends on  $x$ .

“*That John sleeps worries ...*” function  $worry'(sleep'john')$

$$\mathbf{B}fga = f(ga)$$

**B** is Schönfinkel’s **compositor** (he called it **Z**)

These **combinators** get their arguments one at a time:  $\mathbf{B}fga = ((\mathbf{B}f)g)a$

$$\mathbf{B}worry'sleep'john' = worry'(sleep'john')$$

**B**, **C** are invariants; they do the same thing to any function

Schönfinkel (1924) showed that only a handful of combinators can represent any conceivable relation between functions and arguments.

$$\mathbf{I}f = f$$

$$\mathbf{K}fa = f$$

$$\mathbf{T}af = fa$$

$$\mathbf{S}fga = fa(ga) \quad (\text{called fusion by MIS})$$

$$\mathbf{D}fgha = fg(ha)$$

$$\mathbf{\Phi}fgha = f(ga)(ha)$$

Definitive work on combinators: Curry and Feys (1958)

All combinators have two things in common:

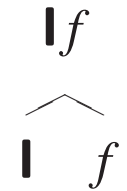
They get their arguments one-at-a-time

They apply the same operation blindly

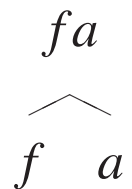
They arise from a **single semantic base**: juxtaposition of  $f$  and  $a$  as  $fa$

Any function-argument relation is binary:  $fabcd = (((fa)b)c)d$

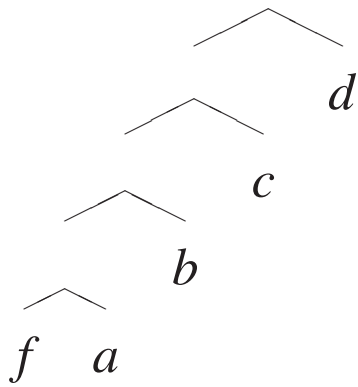
Including zero-argument functions:  $f = \mathbf{I}f$



and one-argument functions:  $fa$



**Currying:** from  $f(a,b,c,d)$  to  $((((fa)b)c)d) = fabcd$  :



**Schönfinkeling** alive in secret sects.

Haskell Brooks Curry 1900 – 1982



## Syntactic Reflex of Combinators

Semantic primitive **juxtaposition** ( $fa$ ) needs a functor and an argument:

$$\mathbf{X}/\mathbf{Y}: f \quad \mathbf{Y}: a \quad \rightarrow \quad \mathbf{X}: fa \quad (\text{>})$$

A non-functor such as  $ba$  is not possible:

$$\mathbf{X}: b \quad \mathbf{Y}: a \quad \rightarrow \quad \mathbf{X}: ba \quad (*\text{>})$$

Semantics says nothing about syntactic order of  $f$  and  $a$ :

$$\mathbf{Y}: a \quad \mathbf{X} \backslash \mathbf{Y}: f \quad \rightarrow \quad \mathbf{X}: fa \quad (\langle)$$

Semantics does say something about which one is syntactically functor:

$$\mathbf{Y}: a \quad \mathbf{X} \backslash \mathbf{Y}: f \quad \rightarrow \quad \mathbf{X}: af \quad (*\langle)$$

$$\mathbf{Y}: f \quad \mathbf{X} \backslash \mathbf{Y}: a \quad \rightarrow \quad \mathbf{X}: fa \quad (*\langle)$$

Semantics says composing functions require syntactic functors:

$$\mathbf{X/Y : f \quad Y/Z : g \quad Z : a \rightarrow X/Z : f(ga)} \quad (>\mathbf{B})$$

$$\mathbf{X/Y : f \quad Y : g \quad Z : a \rightarrow X/Z : f(ga)} \quad (*>\mathbf{B})$$

Semantics does say non-binary combinations are spurious:

$$\mathbf{X/Y : f \quad Y/Z : g \rightarrow X/Z : \lambda x.f(gx)} \quad (>\mathbf{B})$$

Semantics says that syntactic types cannot be arbitrary:

$$\mathbf{X/Y} : f \quad \mathbf{Y/Z} : g \quad \rightarrow \quad \mathbf{X/W} : \lambda x.f(gx) \quad (* > \mathbf{B})$$

Semantics cannot say anything about syntactic order of  $f$  and  $g$ :

$$\mathbf{Y/Z} : g \quad \mathbf{X/Y} : f \quad \rightarrow \quad \mathbf{X/Z} : \lambda x.f(gx) \quad (< \mathbf{B})$$

Syntax says some types are subsumed by others:

$$\mathbf{X}/(\mathbf{X}\backslash\mathbf{Z}): \lambda g.ga \quad \mathbf{X}\backslash\mathbf{Z}: f \quad \rightarrow \quad \mathbf{X}: fa \quad \quad \quad (* > \mathbf{T})$$

But not all:

$$\mathbf{Z}: a \quad \rightarrow \quad \mathbf{X}/(\mathbf{X}\backslash\mathbf{Z}): \lambda f.fa \quad \quad \quad (> \mathbf{T})$$

Consistent directionality of types acts as a projection principle:

$$\mathbf{X}/\mathbf{Y} : f \quad \mathbf{Y}/\mathbf{Z} : g \quad \rightarrow \quad \mathbf{X}/\mathbf{Z} : \lambda x.f(gx) \quad (>\mathbf{B})$$

$$\mathbf{X}/\mathbf{Y} : f \quad \mathbf{Y}/\mathbf{Z} : g \quad \rightarrow \quad \mathbf{X}\backslash\mathbf{Z} : \lambda x.f(gx) \quad (*>\mathbf{B})$$

Having a unique source for semantics (adjacency) spells predictable syntactic machinery

Syntactic machinery does reflect semantic dependencies

Once syntax is established combinatorially, compositional semantics comes for free

every step of the way.

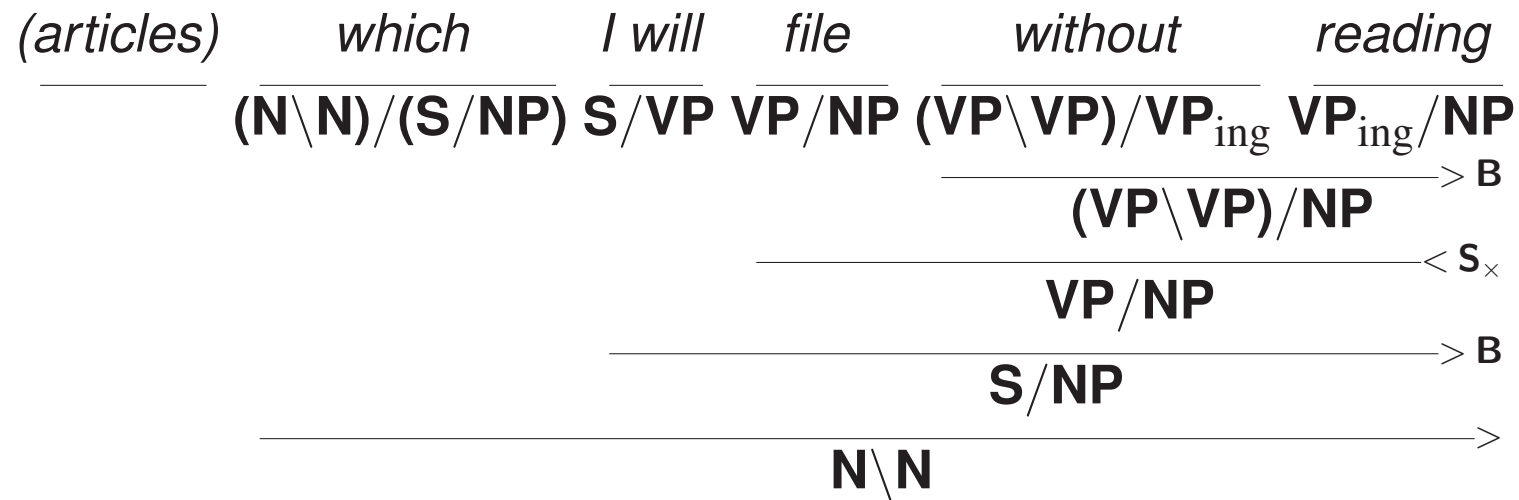
## Adjacency and displacement

John	likes	cats
<b>NP: john'</b>	<b>(S\NP)/NP: likes'</b>	<b>NP: cats'</b>
<b>S\NP: likes' cats'</b> <span style="float: right;">&gt;</span>		
<b>S: likes' cats' john'</b> <span style="float: right;">&lt;</span>		

John	likes	and Mary hates	cats
<b>NP: john'</b>	<b>(S\NP)/NP: likes'</b>		<b>NP: cats'</b>
<b>S/(S\NP): λf.f john'</b> <span style="float: right;">&gt;<sup>T</sup></span>			
<b>S/NP: λx.likes' x john'</b> <span style="float: right;">&gt; B</span>			
⋮			
<b>S: and' (likes' cats' john') (hates' cats' mary')</b> <span style="float: right;">&gt;</span>			



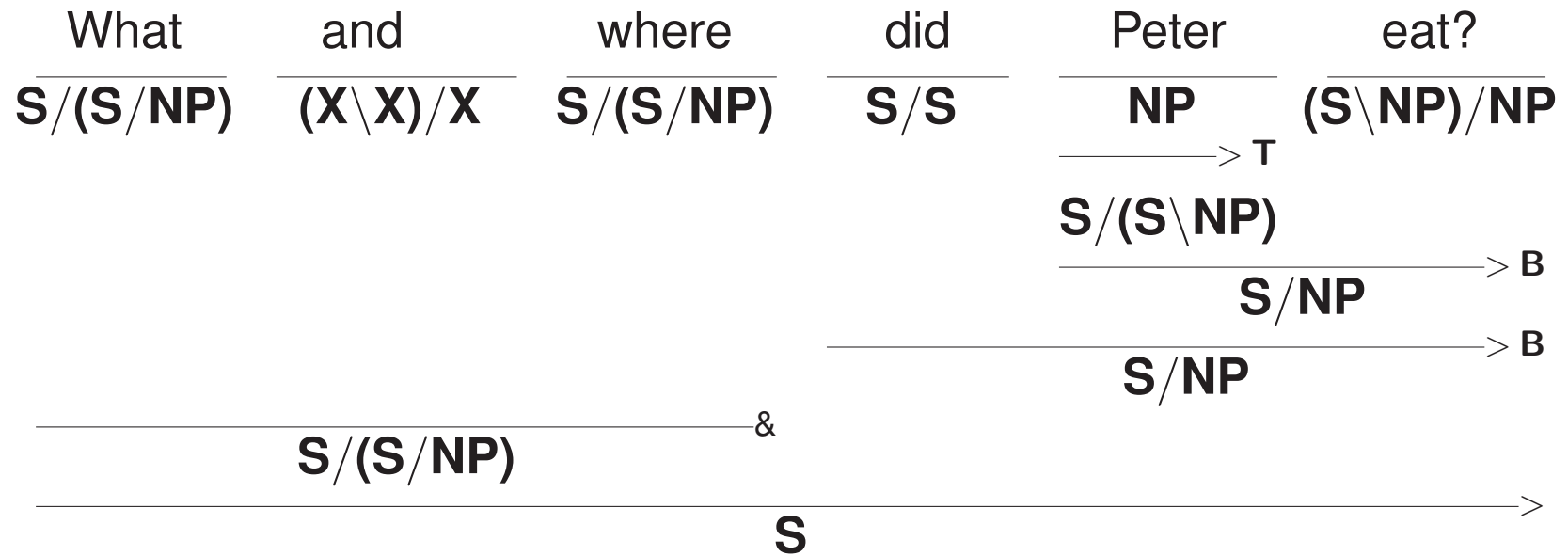
Let  $VP = S \setminus NP$



- (1)a. The cat that [John admires]<sub>S/NP</sub> and [Mary hates]<sub>S/NP</sub>
- b. \*The cat that [John admires]<sub>S/NP</sub> and [bites Mary]<sub>S\NP</sub>
- c. \*The man that [admires John]<sub>S\NP</sub> and [Mary detests]<sub>S/NP</sub>
- d. The man [that admires John]<sub>N\N</sub> and [(that) Mary detests]<sub>N\N</sub>
- e. \*The cat that [John admires]<sub>S/NP</sub> and [Mary hates it]<sub>S</sub>
- f. \*The cat that [John admires it]<sub>S</sub> and [Mary hates]<sub>S/NP</sub>

Bi-clausal Multiple Wh-questions (BMW)

Gracanin-Yüksek (2007)



## **Words versus phrases**

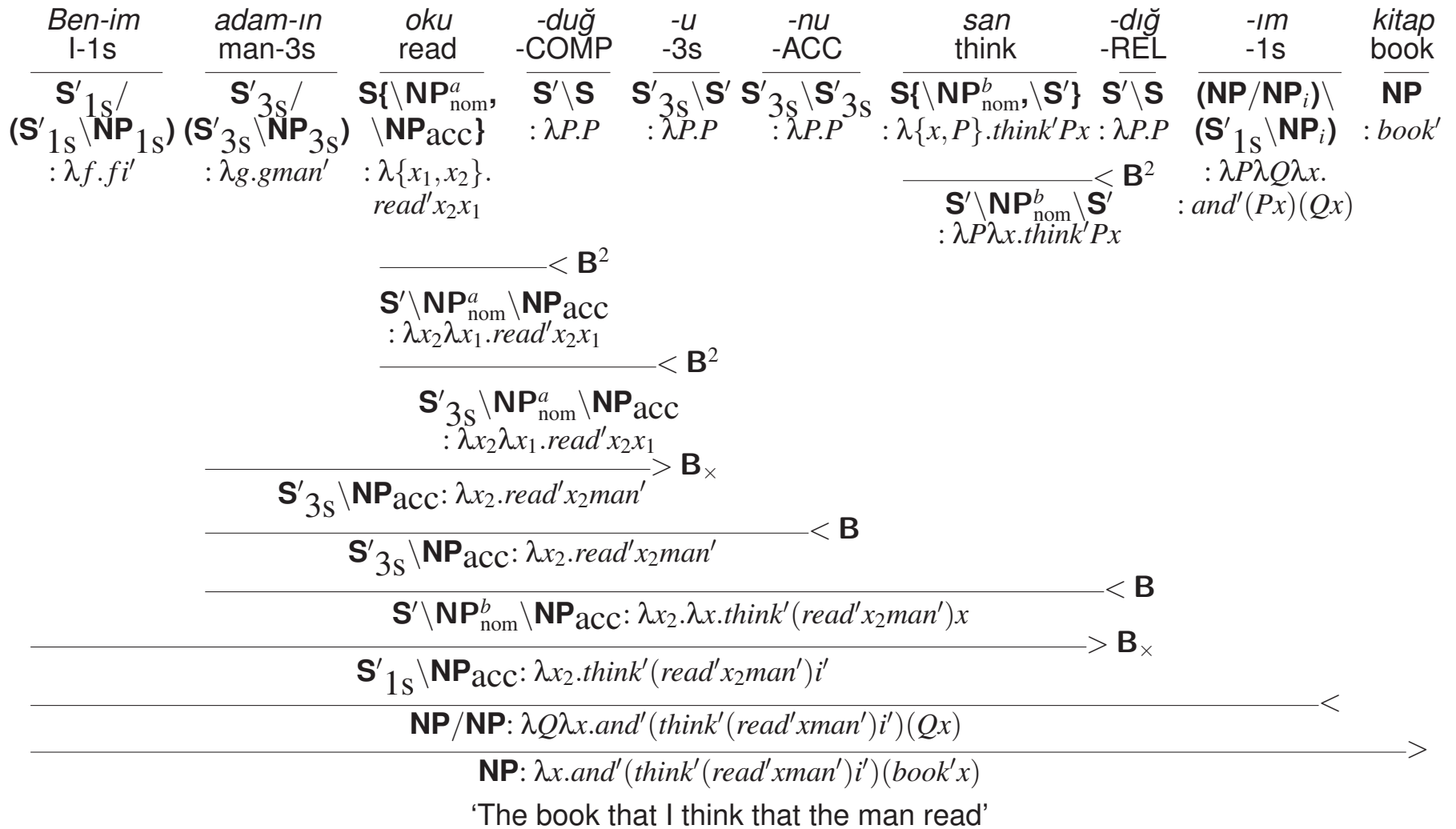
Word grammars

Phrase Grammars

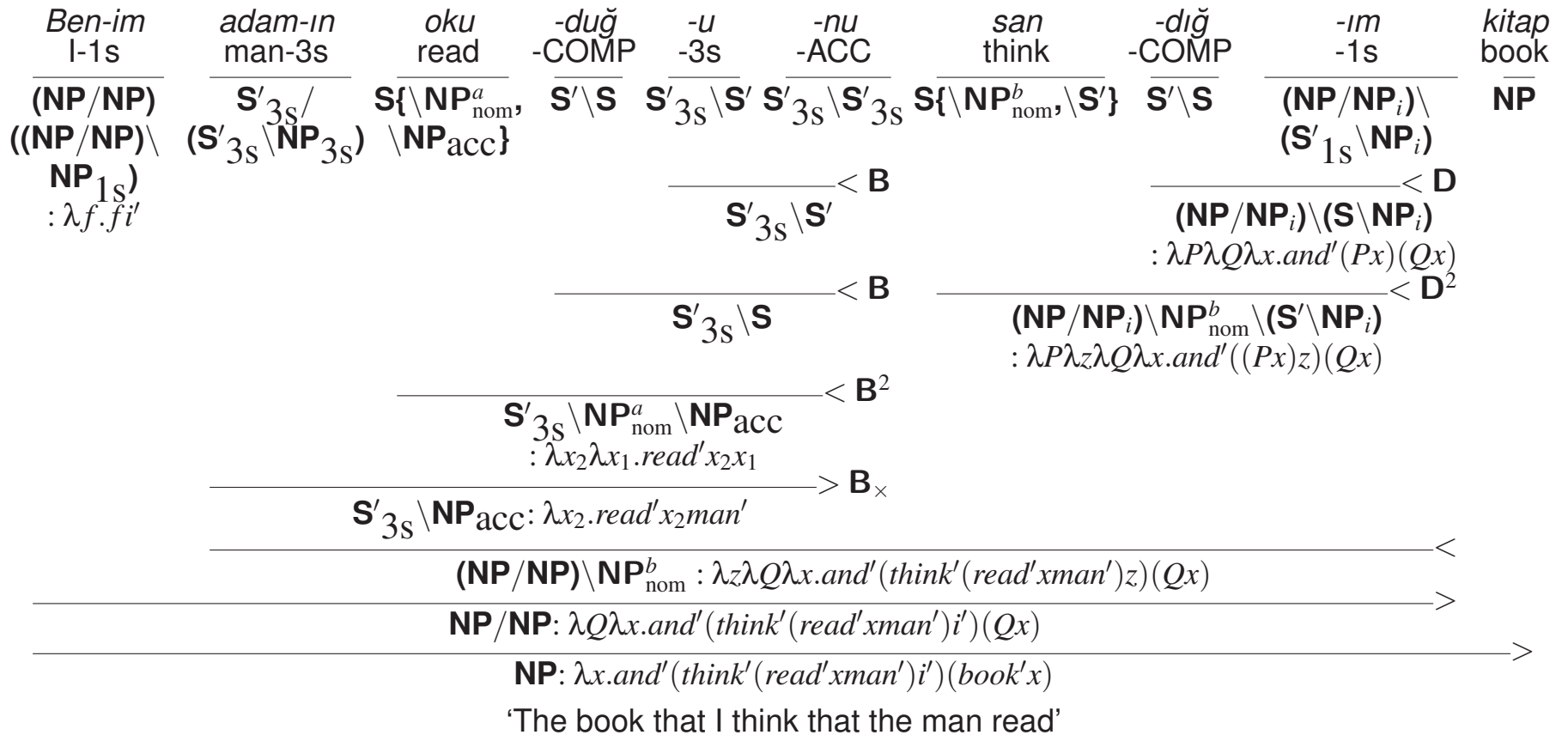
Bridge Grammars

Lexical Integrity

Morphological versus Syntactic Mergers



No respect for word boundaries



All word boundaries are respected

Same lexical assumptions

Same derived semantics

Doing intra-word combinations first before inter-word combinations does not matter combinatorially.

Word/phrase distinction must be an interface problem.



## Summary

Adjacency gave Schönfinkel his foothold into one-at-a-time interpretability of any function/argument configuration.

Polish (function-first) notation and elimination of variables complete the picture of primitive semantic operations as combinators par excellence.

When restricted to bounded use of **B**, **T**, **S**, we get mild context-sensitivity (Joshi, 1985; Vijay-Shanker and Weir, 1993),

Just like human languages.

Their syntactic counterparts are predictable,

due to adjacency and sequential channel.



1889 Dnipropetrovsk, Ukraine – 1942 Moscow

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