SOLVED PROBLEMS - 1
FLOW IN CLOSED CONDUITS
Fully Developed Flows in Closed Conduits

Problem 1 (P 8.3)
The flow of water in a 3-mm diameter pipe is to remain laminar. Plot a graph of the maximum flowrate allowed as a function of temperature for 0<T<100°C. (Hint: find Q for every 20°C intervals)

Solution

For laminar flow \( Re = \frac{V D}{\nu} \leq 2100 \), where \( V = \frac{Q}{A} = \frac{4 Q}{\pi D^2} \)

Thus, the maximum \( Q \) is given by

\[
Re = \left( \frac{4 Q}{\pi \nu D} \right) = \frac{4 Q}{\pi \nu D} = 2100 \]

or

\[
Q = \frac{2100 \pi (0.003 m)}{4} = 4.95 \nu \]

where \( \nu = \frac{m^2}{s} \) and \( Q \sim \frac{m^3}{s} \)

With values of \( \nu \) from Table B.2 we obtain

<table>
<thead>
<tr>
<th>T, deg C</th>
<th>( \nu, \text{m}^2/\text{s} )</th>
<th>( Q, \text{m}^3/\text{s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.79E-06</td>
<td>8.86E-06</td>
</tr>
<tr>
<td>20</td>
<td>1.00E-06</td>
<td>4.95E-06</td>
</tr>
<tr>
<td>40</td>
<td>6.58E-07</td>
<td>3.28E-06</td>
</tr>
<tr>
<td>60</td>
<td>4.75E-07</td>
<td>2.35E-06</td>
</tr>
<tr>
<td>80</td>
<td>3.65E-07</td>
<td>1.81E-06</td>
</tr>
<tr>
<td>100</td>
<td>2.90E-07</td>
<td>1.44E-06</td>
</tr>
</tbody>
</table>

Flowrate vs Temperature
Problem 2 (8.14)
For laminar flow in a round pipe of diameter $D$, at what distance from the centerline is the actual velocity equal to the average velocity?

Solution

For laminar flow $u(r) = V_c \left[ 1 - \left( \frac{2r}{D} \right)^2 \right]$

Average velocity: $V = \frac{V_c}{2}$

\[
\frac{V_c}{2} = V_c \left[ 1 - \left( \frac{2r}{D} \right)^2 \right]
\]

\[
\frac{1}{2} = 1 - \left( \frac{2r}{D} \right)^2
\]

\[
r = \frac{D}{2\sqrt{2}}
\]

\[
r = 0.354D
\]
Problem 3 (8.17)
Glycerine at 20°C flows upward in a vertical 75 mm diameter pipe with a centerline velocity of 1 m/s. Determine the head loss and pressure drop in a 10 m length of the pipe.

For laminar flow in a pipe,
\[ V = \text{average velocity} = \frac{1}{2} V_{\text{max}} = \frac{1}{2} (1 \text{ m/s}) = 0.5 \text{ m/s} \]

Thus,
\[ Re = \frac{D V D}{\mu} = \frac{(1260 \text{ kg/m}^3)(0.5 \text{ m/s})(0.075 \text{ m})}{1.50 \text{ N s/m}^2} = 31.5 < 2100 \]

The flow is laminar so that
\[ V = \frac{(\Delta \rho - \gamma \ell \sin \theta) D^2}{32 \mu} \], where \( \theta = 90^\circ \)

Thus,
\[ \Delta \rho = \frac{32 \mu V D^2}{D^2} + \gamma \ell = \frac{32 (1.50 \text{ N s/m}^2)(10 \text{ m})(0.5 \text{ m})}{(0.075 \text{ m})^2} + (9.81 \text{ m/s}^2)(1260 \text{ kg/m}^3)(10 \text{ m}) \]
\[ = 1.66 \times 10^5 \text{ N/m}^2 \], or \( \Delta \rho = 166 \text{ kPa} \)

Also,
\[ \frac{\rho_1}{\rho} + Z_1 + \frac{V_1^2}{2g} = \frac{\rho_2}{\rho} + Z_2 + \frac{V_2^2}{2g} + h_L \], or with \( V_1 = V_2 \), \( Z_2 - Z_1 = \ell \), and
\[ \rho_1 = \rho_2 + \Delta \rho \] this gives
\[ h_L = \frac{\Delta \rho}{\rho} - \ell = \frac{1.66 \times 10^5 \text{ N/m}^2}{(9.81 \text{ m/s}^2)(1260 \text{ kg/m}^3)} - 10 \text{ m} = 3.43 \text{ m} \]
Problem 4 (8.28)
Water flows downward through a vertical 10 mm diameter galvanized iron pipe with an average velocity of 5 m/s and exits as a free jet. There is a small hole in the pipe 4 m above the outlet. Will water leak out of the pipe through this hole, or will air enter into the pipe through the hole? Repeat the problem if the average velocity is 0.5 m/s.

\[ \frac{u_L^2 + \frac{V^2}{2g} + Z_1}{Z_2 + \frac{V_2^2}{2g}} = \frac{u_L^2}{Z_2} + \frac{V^2}{2g} + Z_2 + \frac{V_2^2}{2g} , \text{ where } \rho_2 = 0, Z_2 = 0, \]

For an average velocity of 5 m/s:

\[ Z_1 = 4 \text{ m}, \quad V_1 = V_2 = V. \]

Thus,

\[ \frac{u_L^2}{Z_2} = \frac{fL}{D} \frac{V^2}{2g} - Z_1, \quad \text{ or } \rho_1 = \frac{fL}{D} \frac{V^2}{2g} - \rho L \]

With \( \epsilon \) from Table 8.1,

\[ \frac{\epsilon}{D} = \frac{0.15 \text{ mm}}{10 \text{ mm}} = 0.015 \]

so that with \( Re = \frac{VD}{\nu} = \frac{(5 \text{ m/s})(0.01 \text{ m})}{1.12 \times 10^{-6} \text{ m}^2/\text{s}} = 4.46 \times 10^4 \)

we obtain \( f = 0.045 \) (see Fig. 8.20).

Thus, from Eq. (1)

\[ \rho_1 = 0.045 \left( \frac{4 \text{ m}}{0.01 \text{ m}} \right)^{1/2} (999 \text{ kg/m}^3)(5 \text{ m/s})^2 - 9800 \frac{N}{m^2} (4 \text{ m}) = 1.86 \times 10^5 \frac{N}{m^2} \]

Since \( \rho_1 > 0 \), water will leak out of the pipe when \( V = 5 \text{ m/s} \).

If \( V = 0.5 \text{ m/s} \), then \( Re = 4.46 \times 10^3 \) and \( f = 0.052 \)

Thus, from Eq. (1)

\[ \rho_1 = 0.052 \left( \frac{4 \text{ m}}{0.01 \text{ m}} \right)^{1/2} (999 \text{ kg/m}^3)(0.5 \text{ m/s})^2 - 9800 \frac{N}{m^2} (4 \text{ m}) = -3.66 \times 10^4 \frac{N}{m^2} \]

Since \( \rho_1 < 0 \), air will enter the pipe when \( V = 0.5 \text{ m/s} \).

Note: The above conclusion is valid regardless of the length, \( l \).
Problem 5 (8.45)

Air flows in a 0.50 m diameter pipe at a rate of 10 m$^3$/s. The pipe diameter changes to 0.75 m through a sudden expansion. Determine the pressure rise across this expansion.

\[
\frac{\rho_1}{2g} + Z_1 = \frac{\rho_2}{2g} + Z_2 + K_L \frac{V_1^2}{2g}, \quad Z_1 = Z_2
\]

and

\[
V_i = \frac{Q}{A_i} = \frac{10 \text{ m}^3}{\text{s}} \frac{1}{\pi (0.50 \text{ m})^2} = 50.9 \text{ m}^2 \text{s}^{-1}, \quad V_2 = \frac{Q}{A_2} = \frac{10 \text{ m}^3}{\pi (0.75 \text{ m})^2} = 22.6 \text{ m}^2 \text{s}^{-1}
\]

Also, for a sudden expansion

\[
K_L = \left(1 - \frac{A_1}{A_2}\right)^2 = \left(1 - \left(\frac{D_1}{D_2}\right)^2\right)^2 = \left(1 - \left(\frac{0.50 \text{ m}}{0.75 \text{ m}}\right)^2\right)^2 = 0.309
\]

Thus, from Eq. (1)

\[
\rho_2 - \rho_1 = \frac{1}{2} \rho (V_i^2 - V_2^2) - K_L \frac{1}{2} \rho V_i^2
\]

\[
= \frac{1}{2} \left(1.23 \text{ kg m}^{-3}\right) \left[\left(50.9 \text{ m}^2 \text{s}^{-1}\right)^2 - \left(22.6 \text{ m}^2 \text{s}^{-1}\right)^2 - (0.309) \left(50.9 \text{ m}^2 \text{s}^{-1}\right)^2\right]
\]

\[
= 1279 \frac{\text{N}}{\text{m}^2} - 492 \frac{\text{N}}{\text{m}^2}
\]

or

\[
\rho_2 - \rho_1 = 787 \frac{\text{N}}{\text{m}^2}
\]

Note: The pressure rise due to the change in speed $(V_2 < V_1)$ is greater than the pressure loss $- 7279 \text{ Pa} > 492 \text{ Pa}$. 

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