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Counting Palladian plans

G Stiny
Centre for Configurational Studies, The Open University, Milton Keynes MK7 6AA, England
W J Mitchell
School of Architecture and Urban Planning, University of California, Los Angeles, California 90024, USA
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Abstract. The parametric shape grammar developed in Stiny and Mitchell (1978) is used to enumerate possible room layouts for Palladian villa plans of fixed sizes. The complete catalogue of room layouts for plans based on 3 x 3 and 5 x 3 grids is given. Members of this catalogue corresponding to Palladio's villa plans in *I Quattro Libri dell'Architettura* (Palladio, 1965) are identified.

In a previous paper (Stiny and Mitchell, 1978) we proposed a parametric shape grammar to generate the complete ground plans of uniaxial Palladian villas. The rules specified in this grammar provide for plans to be constructed in eight stages. The composition of a villa plan takes place in the initial three stages. A bilaterally symmetric, rectangular grid is defined that consists of a $(2m+1) \times n$ $(m \geqslant 0, n > 0)$ array of variously dimensioned rectangular cells, each separated from their immediate neighbours by a standard base-wall thickness. This grid is circumscribed by a rectangle to form the exterior wall of the plan. The interior spaces in this pattern are concatenated recursively to produce an appropriate room layout consisting of rectangular, I-shaped, T-shaped, and +-shaped spaces. The detailing of a room layout so generated is performed in the remaining five stages. Porticos, wall inflections, columns, windows, and doors are added to the basic plan in the Palladian manner to produce complete villa plans. In this way the grammar generates a language of villa ground plans that contains most of the villa projects considered by Palladio in the *Quattro Libri*, as well as a variety of other villa plans in the Palladian style.

In this paper we consider the number of possible room layouts (plans) of a certain size that can be generated by the grammar. The size of a plan is given by the number of columns and rows of rectangular cells in the smallest underlying grid required for its generation. The enumeration of plans of size $(2m+1) \times n$ begins with an *initial plan* consisting of a grid of this size circumscribed by a rectangle. For example, an initial plan of size 5 x 3 is shown in figure 1. The *concatenation rules* of the grammar (rules 12–19 in stage 3) are then applied in a systematic way to produce all distinct plans of the given size. Notice that the enumeration of plans does not depend on the

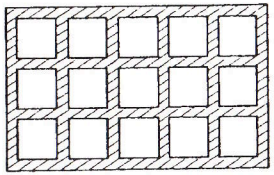

Figure 1. An initial plan of size 5 x 3.

dimensions of the interior spaces in the initial plan. In enumerating plans, we are interested in the number of distinct room arrangements that can be derived from the initial plan and not in the actual dimensions of these spaces.

An initial plan of given size contains a finite number of component cells. The application of each of the concatenation rules to a plan reduces the number of its interior spaces to produce a new plan. Consequently the number of plans that can be derived from the initial plan by applying the concatenation rules recursively is limited.

The exact number of plans that can be generated from the initial plan may be determined in this way. We begin with a series of plans consisting of the initial plan only. To the first plan in the series constructed so far to which we have not tried to apply the concatenation rules, we apply each of these rules that can be applied. Of the new plans so produced, we agree to add only those to the end of the series that are distinct from plans already occurring in the series. Two plans are *distinct* if and only if they are not identical or one can not be obtained from the other by rotating it 180 degrees. Further, we add only those plans that can not be generated from a smaller initial plan, that is, one with a smaller number of columns or rows. We continue this process until no more plans can be added to the series. The final series so constructed contains all of the distinct plans that can be derived from the initial plan and from no smaller initial plan.

Table 1 indicates the number of plans generated from initial plans of the sizes used by Palladio for his villa projects in the *Quattro Libri*.

The complete enumeration of the plans derived from an initial plan of size 3 x 3 is given in section 1 of the appendix and from an initial plan of size 5 x 3 in section 2. In each series, the initial plan is the first enumerated. This plan consists of a 3 x 3 or 5 x 3 array of square spaces. Other plans are formed by combining these spaces in the appropriate way. The richness of the concatenation rules used to generate these plans is amply demonstrated by the diversity of the plans found in this catalogue. Indeed this recursive characterization of Palladio's system of room layout provides considerable opportunity to introduce variety into villa plans even at this basic compositional stage. Reliance on detailing to distinguish villa projects hardly seems necessary.

One of Palladio's villa ground plans in Book 2 of the *Quattro Libri* is generated from an initial plan of size 3 x 3; eight are generated from initial plans of the most frequently used size, 5 x 3. The room layouts used in each of these projects are drawn to a common scale in figure 2. The Roman numeral in capital letters beneath each plan refers to the plate number of the corresponding villa project in Book 2 of the Dover reprint edition of Isaac Ware's 1738 English translation of the *Quattro Libri*

Table 1. The number of plans generated from initial plans of various sizes.

Plan size	Number of plans				
	containing rectangular rooms only	containing one central I-shaped room	containing one central T-shaped room	containing one central +-shaped room	total
3 x 3	12	1	6	1	20
5 x 3	119	5	78	8	210
5 x 4	1132	59	1039	136	2366
5 x 5	10192	704	12092	2029	25017
5 x 6	92664	7748	132120	25283	257815
5 x 7	836033	82869	1393116	292773	2604791
7 x 3	1114	31	780	65	1990
7 x 4	27916	1205	27110	3388	59619
7 x 5	644460	38307	791328	126222	1600317

(Palladio, 1965). In figure 2(a) the Arabic number identifies the corresponding plan in section 1 of the appendix, and in figure 2(b) the corresponding plan in section 2. The plans in figure 2 and their counterparts in the appendix differ only in the way their interior spaces have been dimensioned. Notice that the plan in figure 2(a) contains rectangular spaces only. Also five of the plans in figure 2(b) contain rectangular spaces only. The interior spaces in two of these plans are arranged in the same way. One plan in figure 2(b) contains a central I-shaped room, one a central T-shaped room, and one a central +-shaped room.

To our eyes most of the plans enumerated in the appendix look Palladian. Only those in which more than one interior space or part of an interior space extends the full length or width of the plan, for example, plans 6, 12, and 19 in section 1 and plans 33, 85, and 190 in section 2, seem marginal as Palladian compositions. If desired

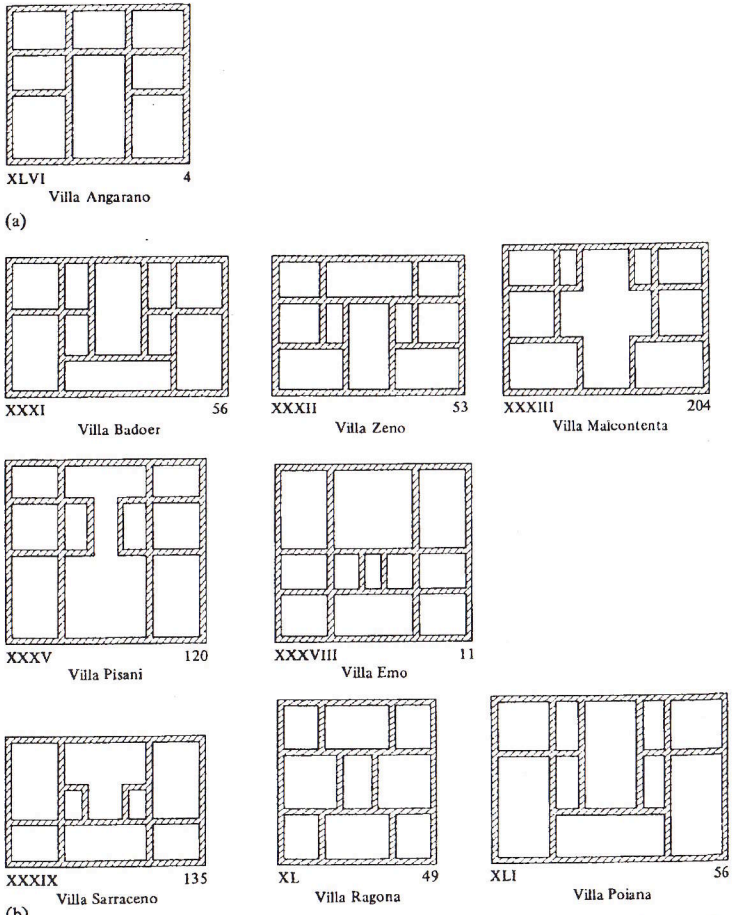

Figure 2. The room layouts of size (a) 3 x 3, and (b) 5 x 3 corresponding to Palladio's villa ground plans.

these plans may be excluded from the catalogue by modifying the concatenation rules in the grammar used to generate them. These changes would introduce additional labels to prevent such spaces from being formed. Of course some readers may wear different glasses. They may well wish to specify their own rules to generate Palladian plans.

Acknowledgements. We wish to thank C F Earl for helping with the original hand enumeration of plans of size 3 x 3 and 5 x 3. R Krishnamurti is responsible for the computer algorithm used to enumerate plans of larger size.

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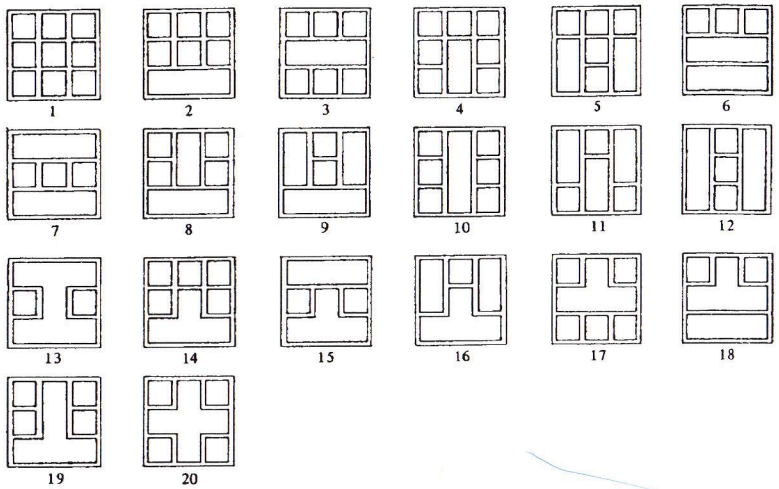
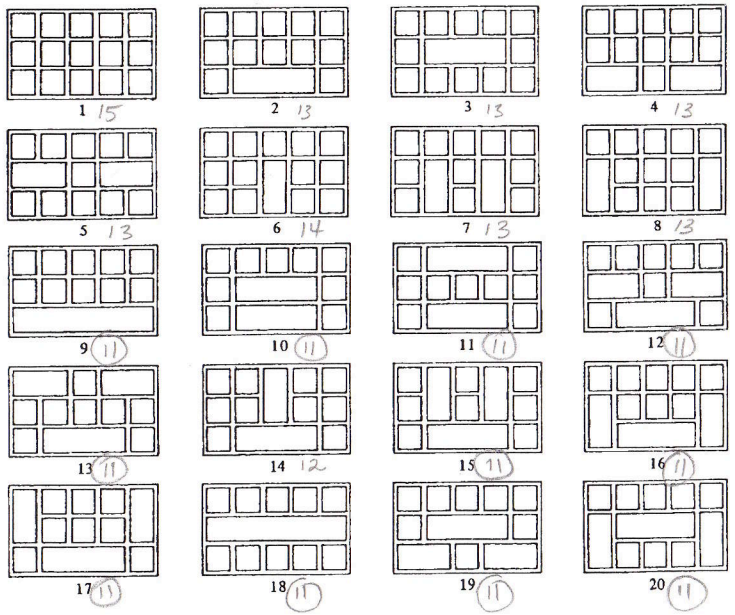
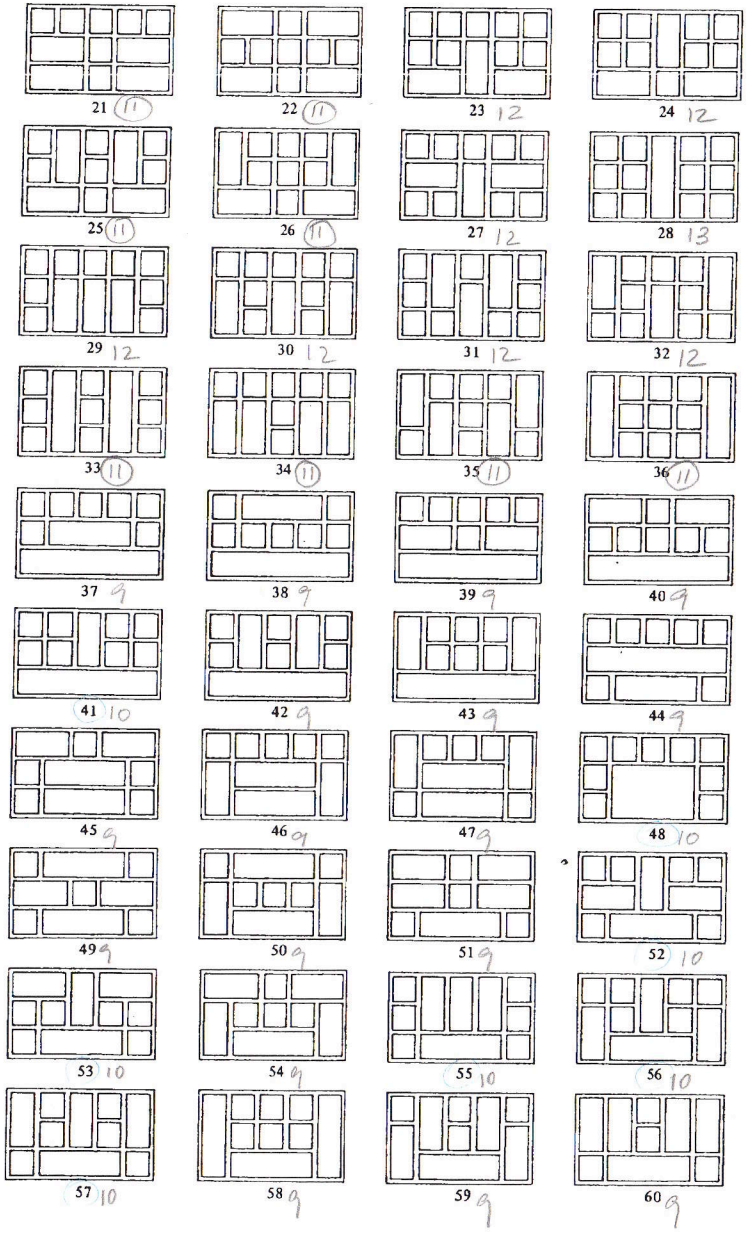
APPENDIX: A catalogue of possible room layouts for Palladian villas

1 The enumeration of all plans of size 3 x 3



2 The enumeration of all plans of size 5 x 3

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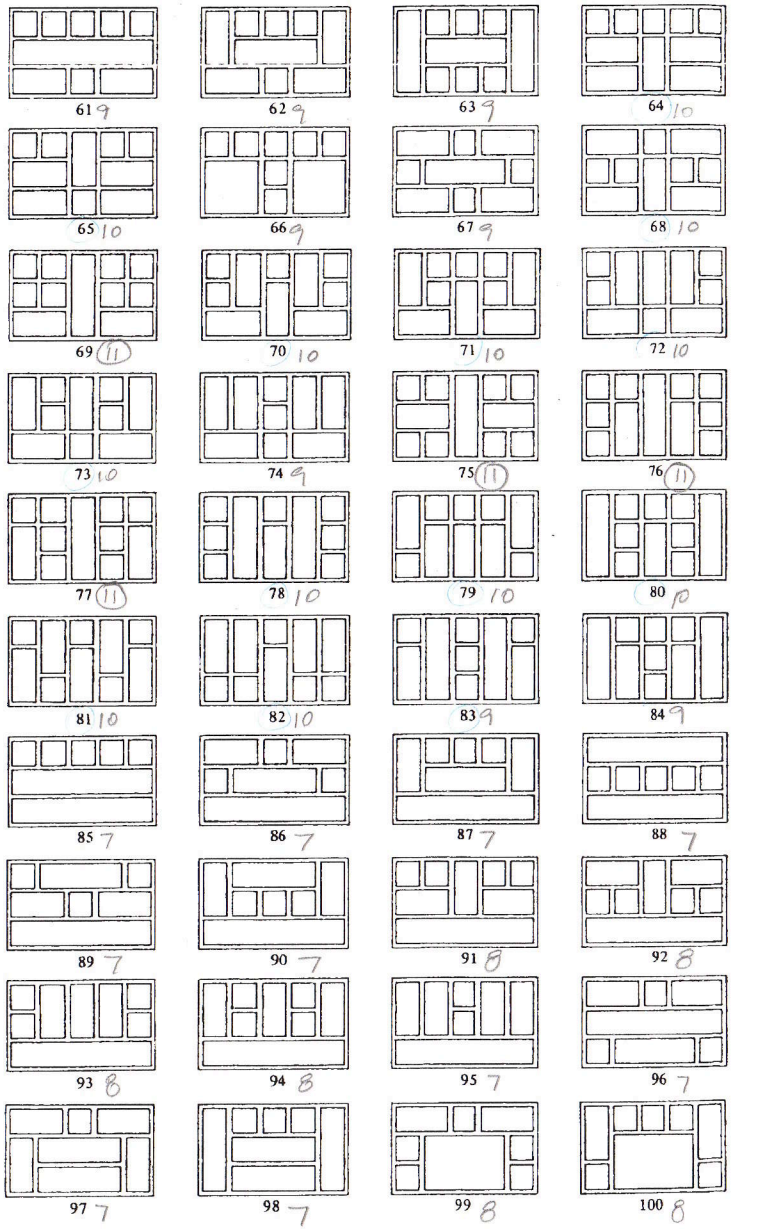
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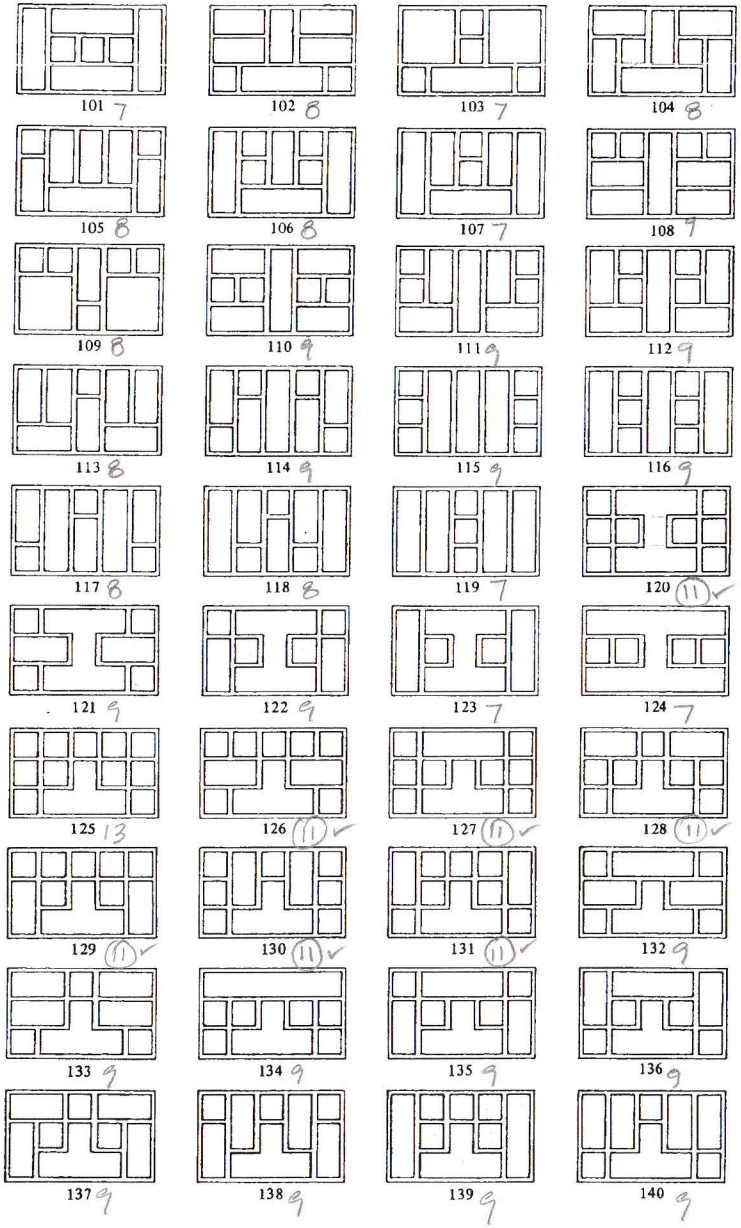
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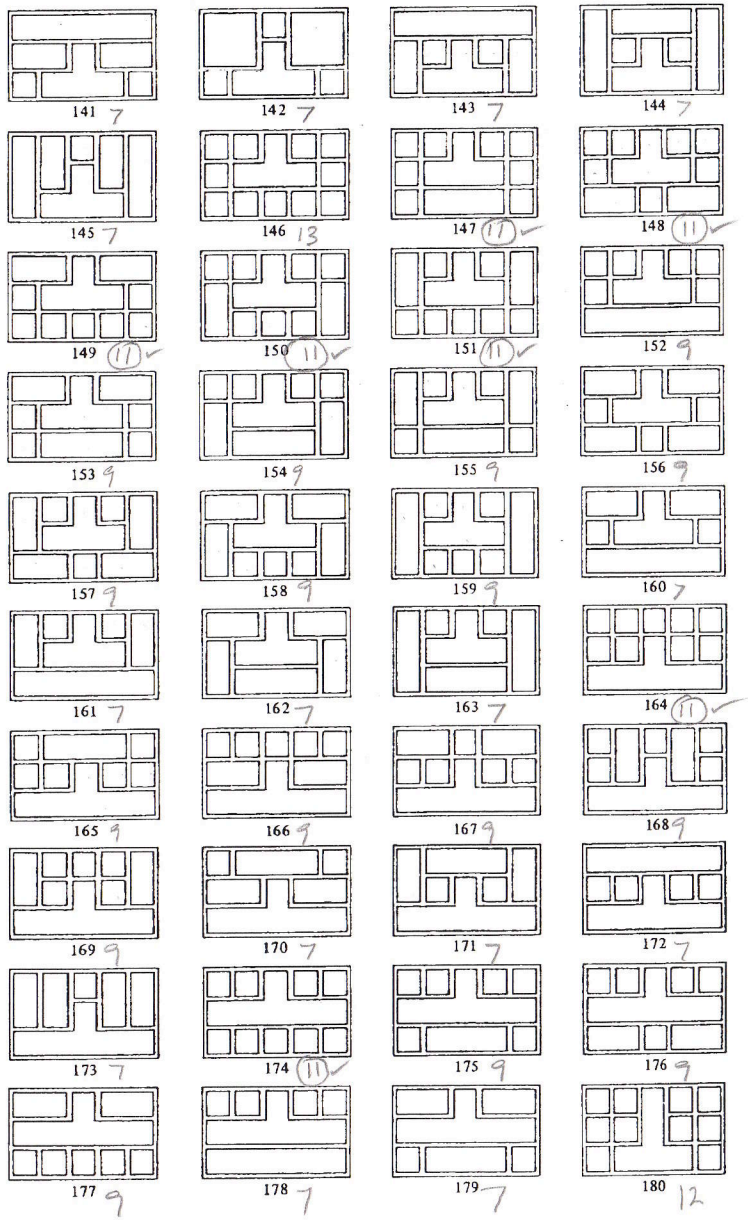
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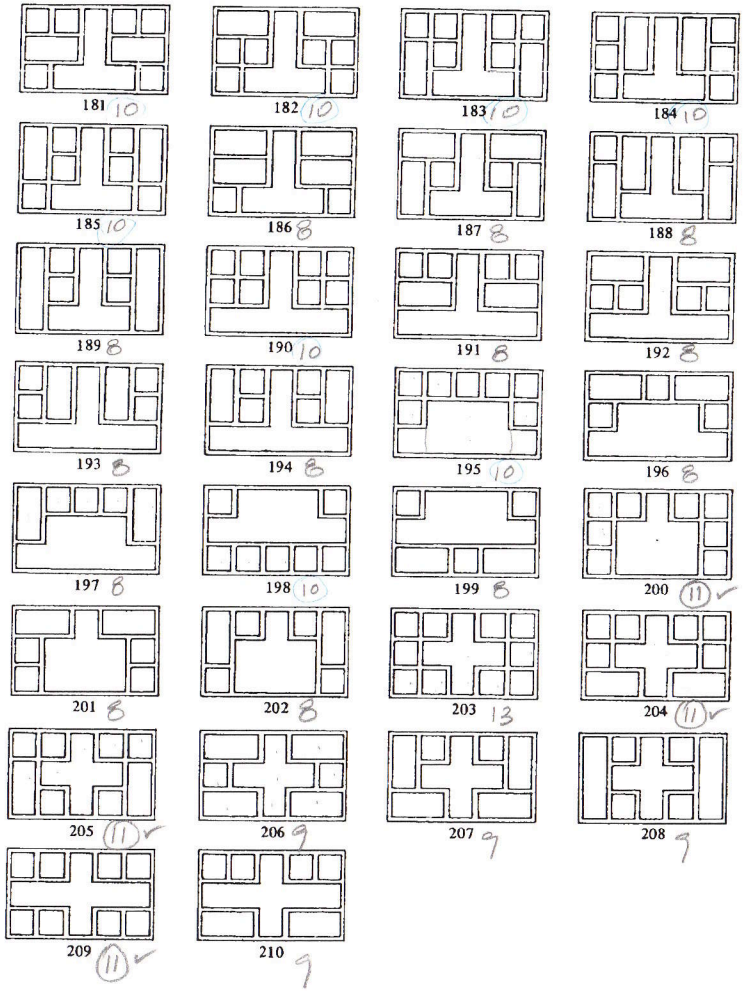

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An evaluation of Palladian plans

G Stiny
Centre for Configurational Studies, The Open University, Milton Keynes MK7 6AA, England
J Gips
Computer Science Department, School of Management, Boston College, Chestnut Hill, Massachusetts 02167, USA
Received 16 October 1978

Abstract. Criteria are suggested for the aesthetic evaluation of Palladian villa plans. These criteria are applied to two catalogues of all possible plans constructed on underlying grids of sizes 3 x 3 and 5 x 3 respectively.

The question of aesthetic value immediately arises when we examine two or more architectural objects, for example, buildings, plans, or elevations, in the same style. How can these objects be ordered in terms of their quality? There is no single, absolute answer to this question. Any ordering depends on the conventions and criteria used by the observer. Two different knowledgeable observers may disagree about which objects in the same style are better than others because they may apply different evaluative criteria.

Consider the two catalogues (series) of Palladian villa plans enumerated by Stiny and Mitchell (1978b). These catalogues contain all possible distinct plans based on underlying grids of sizes 3 x 3 and 5 x 3 that can be generated by use of the parametric shape grammar developed in Stiny and Mitchell (1978a). The grammar produces the ground plans of villas in the Palladian style. The first catalogue contains the 20 possible distinct plans of size 3 x 3, the second the 210 of size 5 x 3. One of Palladio's villa projects in *I Quattro Libri dell'Architettura* (Palladio, 1965) is based on a 3 x 3 plan and thus occurs in the first catalogue; eight are based on 5 x 3 plans and thus occur in the second catalogue.

In this paper, we propose and investigate a set of evaluative criteria for such catalogues of plans. These criteria are of two types. Criteria of the first type divide a catalogue into two groups. The smaller group includes those plans used by Palladio in his villa projects as well as a number of other plans. The larger group contains no plans used by Palladio. The criteria of the second type order the plans in a given group in terms of an evaluation measure that we have developed for other art forms (Stiny and Gips, 1978). This measure determines the aesthetic value of an object based on a relationship between the way it is generated by a given procedure, for example, a shape grammar, and the way it is described.

The evaluative criteria considered do not depend on the actual dimensions of rooms in plans and hence do not incorporate any system of proportions. Rather, the evaluative criteria are based on the arrangements of rooms in plans and the shapes and sizes of these rooms in terms of an underlying grid. Aesthetic discussions of proportions can be found elsewhere.

Partitioning a catalogue of plans

A catalogue of plans of a given size is partitioned into two groups by applying two provisos. These rules were obtained by careful examination of the plans actually