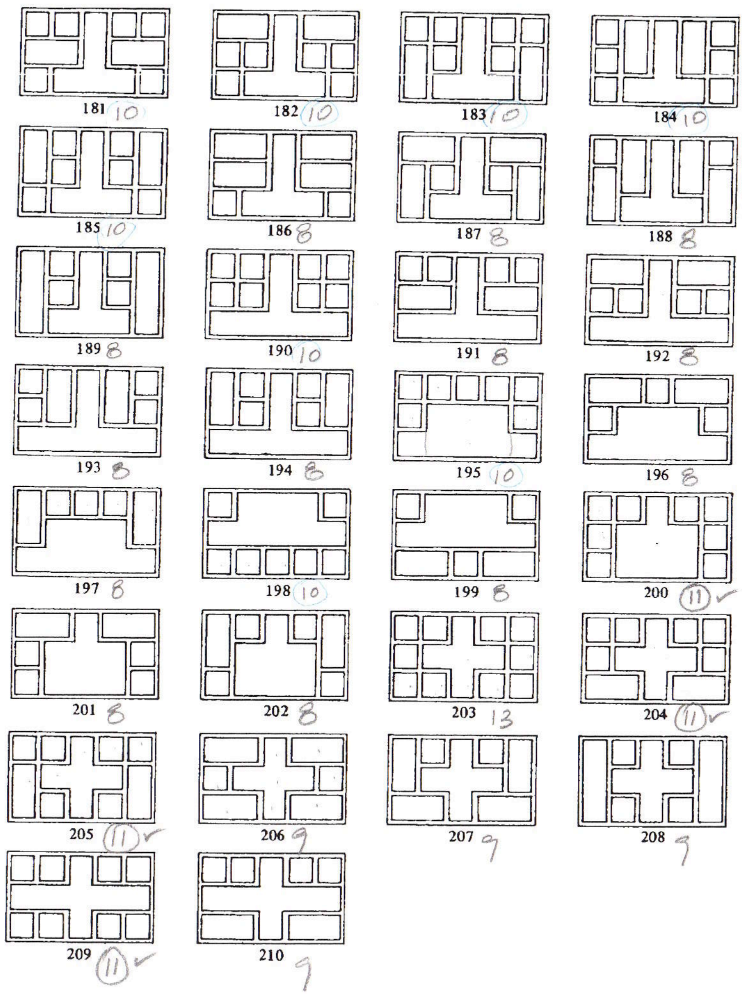

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An evaluation of Palladian plans

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Abstract. Criteria are suggested for the aesthetic evaluation of Palladian villa plans. These criteria are applied to two catalogues of all possible plans constructed on underlying grids of sizes 3 x 3 and 5 x 3 respectively.

The question of aesthetic value immediately arises when we examine two or more architectural objects, for example, buildings, plans, or elevations, in the same style. How can these objects be ordered in terms of their quality? There is no single, absolute answer to this question. Any ordering depends on the conventions and criteria used by the observer. Two different knowledgeable observers may disagree about which objects in the same style are better than others because they may apply different evaluative criteria.

Consider the two catalogues (series) of Palladian villa plans enumerated by Stiny and Mitchell (1978b). These catalogues contain all possible distinct plans based on underlying grids of sizes 3 x 3 and 5 x 3 that can be generated by use of the parametric shape grammar developed in Stiny and Mitchell (1978a). The grammar produces the ground plans of villas in the Palladian style. The first catalogue contains the 20 possible distinct plans of size 3 x 3, the second the 210 of size 5 x 3. One of Palladio's villa projects in *I Quattro Libri dell'Architettura* (Palladio, 1965) is based on a 3 x 3 plan and thus occurs in the first catalogue; eight are based on 5 x 3 plans and thus occur in the second catalogue.

In this paper, we propose and investigate a set of evaluative criteria for such catalogues of plans. These criteria are of two types. Criteria of the first type divide a catalogue into two groups. The smaller group includes those plans used by Palladio in his villa projects as well as a number of other plans. The larger group contains no plans used by Palladio. The criteria of the second type order the plans in a given group in terms of an evaluation measure that we have developed for other art forms (Stiny and Gips, 1978). This measure determines the aesthetic value of an object based on a relationship between the way it is generated by a given procedure, for example, a shape grammar, and the way it is described.

The evaluative criteria considered do not depend on the actual dimensions of rooms in plans and hence do not incorporate any system of proportions. Rather, the evaluative criteria are based on the arrangements of rooms in plans and the shapes and sizes of these rooms in terms of an underlying grid. Aesthetic discussions of proportions can be found elsewhere.

Partitioning a catalogue of plans

A catalogue of plans of a given size is partitioned into two groups by applying two provisos. These rules were obtained by careful examination of the plans actually used by Palladio for his villa projects in the *Quattro Libri*. Both provisos emphasize the importance of the central room(s) in plans.

1. Any part of a room that extends from one exterior wall to the opposite exterior wall must lie along the north–south axis of symmetry of the plan, where 'north–south' is fixed by the orientation of the plan on the page.
2. The exterior room(s) cut by the north–south axis of symmetry must be as large as any other rooms in the plan, where the size of a room is given by the number of grid cells it contains and not by its actual physical dimensions.

Plans that satisfy both provisos are in one group. This group contains all of the plans of the given size used by Palladio in his villa projects. Plans that satisfy either one or neither of the provisos are in the second group.

The first proviso ensures that only the central room in a plan can extend from the north exterior wall to the south exterior wall and that no room can extend from the east exterior wall to the west exterior wall. Figure 1(a) shows three plans that satisfy this proviso, and figure 1(b) shows three plans that do not.

The second proviso ensures that the exterior room(s) on the north–south axis of symmetry are among the largest in the plan. Figure 2(a) shows three plans that satisfy this proviso, and figure 2(b) shows three plans that fail to do so.

The three plans of size 3 x 3 and the fifty-seven plans of size 5 x 3 that satisfy both provisos are shown in the last section of this paper.

A third proviso for partitioning catalogues of plans was postulated.

3. Any grouping of four cells, in the underlying grid for a plan, that forms a 'square' must not be such that all of the cells occur in rooms of the same shape, where the shape of a room is given by the arrangement of the grid cells it contains and not by its actual physical dimensions.

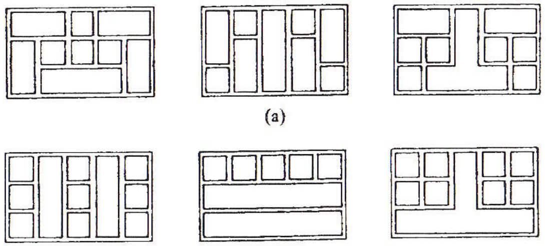


Figure 1. Plans shown in (a) satisfy proviso 1; plans in (b) fail to do so.

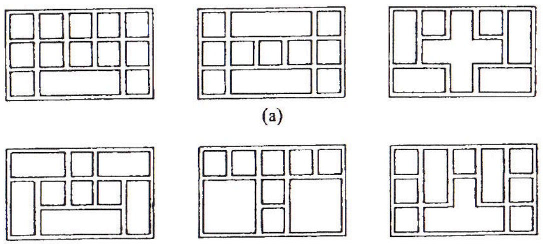


Figure 2. Plans shown in (a) satisfy proviso 2; plans in (b) fail to do so.

Figure 3(a) shows three plans that satisy this additional proviso, and figure 3(b) shows three plans that fail to do so. The asterisks show the groupings of cells that prevent the plans in figure 3(b) from satisfying the proviso.

All plans of sizes 3 x 3 and 5 x 3 used by Palladio satisfy the third proviso. However, some of his villa projects based on plans of larger sizes fail to do so. Figure 4 shows two such plans. Even though we would have liked to have used this proviso, it was rejected on empirical grounds. It should be noted that all of the plans used for the villa projects in the *Quattro Libri*, no matter what their sizes, satisfy the first two provisos.

Provisos 1 and 2 constitute the evaluative criteria used for distinguishing villa plans that are more desirable in Palladian terms from those that are less desirable. Alternatively these provisos could have been incorporated in the rules of the grammar used to generate plans. This approach was not taken because all of the plans enumerated in Stiny and Mitchell (1978b) appear to our eyes to be basically Palladian.

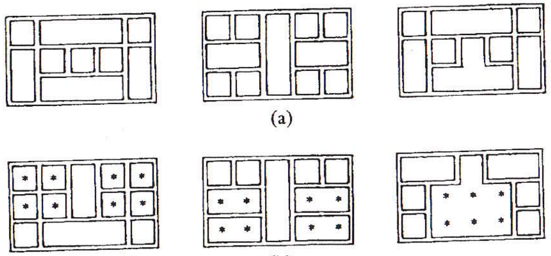


Figure 3. Plans shown in (a) satisfy proviso 3; plans in (b) fail to do so.

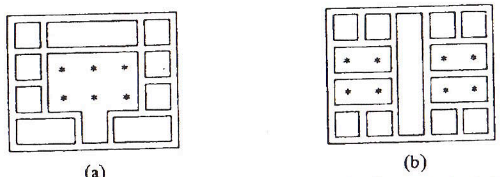


Figure 4. Two plans used by Palladio in his villa projects that fail to satisfy proviso 3. Plan (a) is used in the Villa Cornaro, and plan (b) in the Villa Moncenigo.

Evaluating individual plans

We now consider a method for assigning aesthetic values to individual plans. This method is applied to plans satisfying provisos 1 and 2.

We would have liked to have developed a measure that assigned high aesthetic values to all of the plans actually used by Palladio in his villa projects. After considerable effort, this goal proved elusive. However, the measure presented is interesting in its own right.

In Stiny and Gips (1978), we proposed and investigated an evaluation measure which we named E_Z. This measure assigns aesthetic values to objects of a certain type or style in terms of their descriptions and the rules used to generate them. Here the description of an object is taken to be a one-by-one specification of its essential features and properties. The aesthetic value of the object is given by the ratio of the length of the description of the object to the length of the information required to generate it by use of some fixed procedure. This measure assigns high aesthetic values to objects with lengthy descriptions that can be parsimoniously generated. The measure was shown to have a straightforward interpretation in terms of the standard aesthetic canon of 'unity and variety' and also in terms of an algorithmic formulation of information theory.

To apply the evaluation measure E_Z to calculate the aesthetic value for an individual Palladian plan, we use (1) a sequence of symbols to describe the plan and (2) a sequence of symbols to specify the information needed to generate the plan. The aesthetic value assigned to the plan is the ratio of the lengths of these two sequences.

Describing plans

A plan is here described by a *shape table* and an *occurrence table.*

The shape table lists each distinct shape in the plan that contains more than one cell. For example, the plan shown in figure 5(a), corresponding to the room layout used by Palladio in the Villa Malcontenta, contains two distinct multicellular shapes: a cross consisting of five cells and a rectangle consisting of two cells. A pictorial representation of the shape table for this plan is shown in figure 5(b). Each shape in the shape table is represented symbolically by associating coordinates with its component cells and listing these in lexicographic order. For example, the symbolic representations of the cross and the rectangle in figure 5(b) are

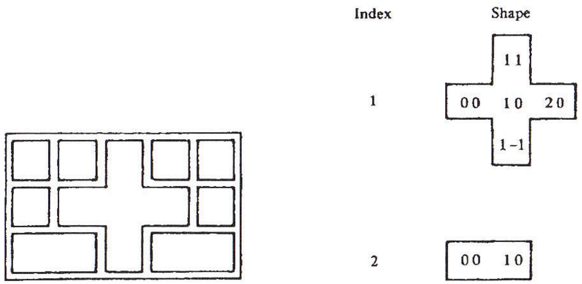
00 10 11 1−1 20   and   00 10

respectively (the different spacings between the numbers are merely to improve readability). The symbolic representation of the shape table is formed by the concatenation of the symbolic representations of the shapes occurring in it in the order given by their indices. Thus the shape table for the plan used in the Villa Malcontenta has the symbolic representation

00 10 11 1−1 20 00 10.

Because the sequence of symbols corresponding to each shape always begins with 00, the indices of shapes need not be given explicitly.

The occurrence table for a plan indicates the location of each multicellular shape in the plan. For example, the occurrence table for the Villa Malcontenta indicates where a cross occurs in its plan and where a rectangle occurs. There is one entry in the occurrence table for each multicellular room in a plan. Each entry consists of four numbers. The first is the index of a shape in the associated shape table. The second and third indicate the coordinates in the plan of the 00 cell in the shape



(a) (b)

Figure 5. The shape table (b) for the plan (a) used in the Villa Malcontenta.

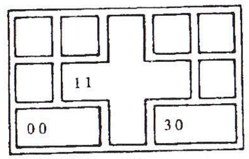
with this index in the shape table. The fourth number is 1 or 0, indicating whether or not the shape should be rotated 90°. The occurrence table for the Villa Malcontenta is given in figure 6(a). The first entry indicates the exact location of the cross in the plan [see figure 6(b)]; the second and third entries indicate the exact locations of the rectangle. The symbolic representation of the occurrence table is formed by concatenating its entries. The occurrence table for the plan used in the Villa Malcontenta has the symbolic representation

1 1 1 0 2 0 0 0 2 3 0 0.

A plan is completely described by giving both its shape table and occurrence table. Notice that unicellular rooms are not specified explicitly. All spaces in the plan not accounted for by the shape table and occurrence table are assumed to be unicellular rooms and hence are fixed by the underlying grid.

Shape index	Coordinates of 0 0 cell	Rotation
1	1 1	0
2	0 0	0
2	3 0	0

(a)



(b)

Figure 6. The occurrence table (a) for the plan used in the Villa Malcontenta, and (b) a pictorial representation illustrating the construction of the table.

Generating plans

The procedure for generating plans consists of the room-layout rules (stage 3, rules 12–19) in the parametric shape grammar given in Stiny and Mitchell (1978a). A plan of a certain size is generated by recursively applying a sequence of these rules to a grid of this size circumscribed by a rectangle. The information needed to specify the generation of the plan is given by indicating the rules in this sequence and the places they apply. For example, the plan for the Villa Malcontenta is produced by applying the sequence of rules: 13, 12, and 18. [See Stiny and Mitchell (1978a) for the actual statement and application of these rules.] The place where each of these rules applies is given by two coordinates which associate part of the left-hand side of the rule with a particular cell position in the plan. Hence each rule application is specified by three numbers: one indicating the rule and two indicating the place it is applied. The information needed to specify the generation of the plan of the Villa Malcontenta is given by a sequence of nine numbers, three for each rule application. Additionally, in plans consisting of rectangular rooms only, rule 19 must be applied to terminate the generation. The application of this final rule need be indicated by its rule number only.

Evaluating plans

The evaluation measure E_Z can now be used to compute the aesthetic value of any plan.

The length of the description of a plan is the sum of the number of numbers representing its shape table and the number of numbers representing its occurrence table. This value may be viewed as a measure of the *visual complexity* of a plan. The length of the description of the plan used for the Villa Malcontenta is $14+12 = 26$. In general, the length of a description is twice the total number of cells in the shapes in the shape table plus four times the number of entries in the occurrence table.

The length of the information required to generate a plan is the number of numbers required to specify the sequence of rule applications. This value may be considered as a measure of the *specificational simplicity* of a plan. The length of the information needed to specify the generation of the plan used for the Villa Malcontenta is 9. In general, the length of the information required to generate a plan containing rectangular rooms only is three times the number of nonterminating rules applied plus one; otherwise the length is three times the number of rules applied.

The aesthetic value for a plan is the ratio of the length of its description to the length of the information used to generate it. The aesthetic value for the plan used for the Villa Malcontenta is 26 divided by 9, which equals 2·89.

A plan is said to be aesthetically superior to another if it is assigned a higher aesthetic value.

The aesthetic values assigned to the plans of sizes 3 x 3 and 5 x 3 satisfying provisos 1 and 2 are shown in figures 7 and 8 respectively. In both figures the plans are listed in order of decreasing aesthetic value. The first number under a plan is its catalogue number in Stiny and Mitchell (1978b). The second number is its aesthetic value computed by the evaluation measure E_Z. An asterisk distinguishes those plans

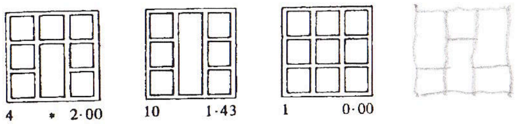


Figure 7. Plans of size 3 x 3 satisfying provisos 1 and 2, listed in order of decreasing aesthetic value.

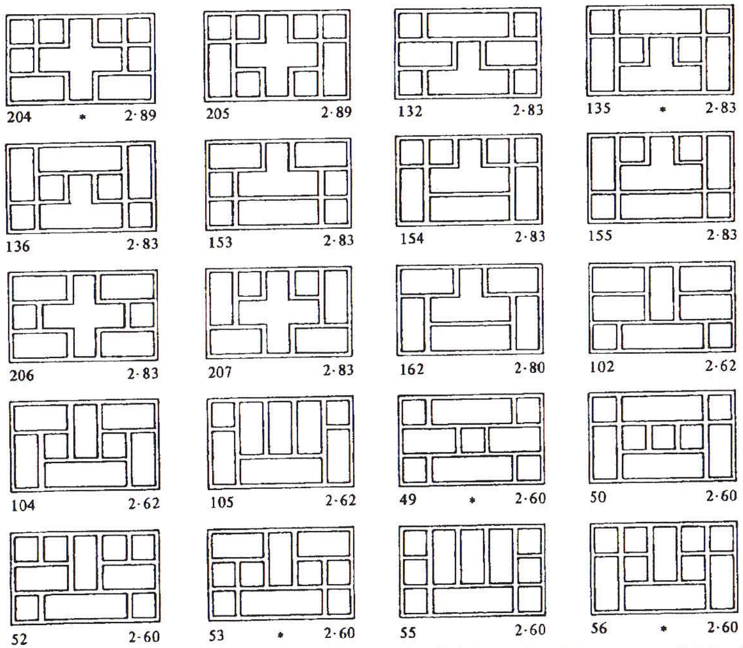


Figure 8. Plans of size 5 x 3 satisfying provisos 1 and 2, listed in order of decreasing aesthetic value.

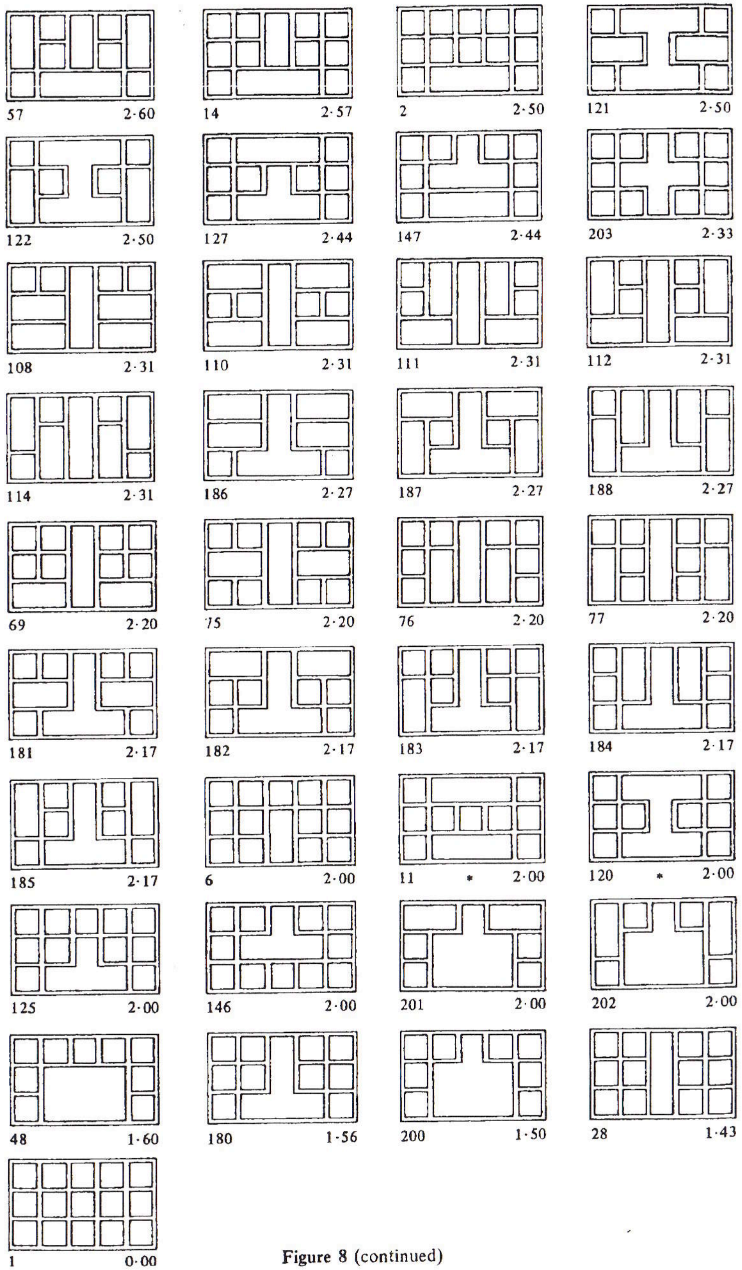


Figure 8 (continued)

used by Palladio in his villa projects. In figure 7, plan 4 is used in the Villa Angarano; in figure 8, plan 204 is used in the Villa Malcontenta, plan 135 in the Villa Sarraceno, plan 49 in the Villa Ragona, plan 53 in the Villa Zeno, plan 56 in the Villa Badoer and the Villa Poiana, plan 11 in the Villa Emo, and plan 120 in the Villa Pisani.

The three plans of size 3 x 3 in figure 7 are all assigned different aesthetic values by the evaluation measure E_Z. The plan used by Palladio ranks first. The fifty-seven plans of size 5 x 3 in figure 8 are assigned nineteen different aesthetic values by the evaluation measure E_Z. In terms of these nineteen values, the plans used by Palladio rank first, second, fifth, and fourteenth. Even though many of the plans used by Palladio in his villa projects rank high in terms of the conventions embodied by the evaluation measure E_Z, the reader is reminded that these conventions are not intended to reflect Palladio's own aesthetic preferences. Many other conventions, and corresponding orderings, are possible.

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The counting of rectangular dissections

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Abstract. In this paper results are presented from two independently developed computer programs—algorithms RK and CB—on counting and classifying rectangular dissections. A population census is given for all weights less than eleven. In spite of the radically different approaches adopted by the two algorithms in solving this enumeration problem, both sets of results agree completely.

Definitions

Rectangular gratings, defined in a mathematical sense by Newman (1964) and first referred to in the design literature by March (1972), form the basis of the representation of rectangular dissections which is discussed in this paper. In the present context they have been elaborated upon by Mitchell et al (1976) and by Bloch (1976).

An (l, m) rectangular grating is formed by drawing $l-1$ straight line segments parallel to one side of a rectangle and $m-1$ such segments parallel to an adjacent side. The lines go right across the rectangle in such a manner that segmentation divides it into edge-connected rectangular cells, or *two-cells*, arranged in l rows and m columns. Two rectangular gratings will be defined to be equivalent if they have the same number of rows and columns. This is an equivalence relation, and the (l, m) equivalence class, for example, consists of all rectangular gratings with l rows and m columns, irrespective of the different spacings of the line segments. Select as representative of each class the grating in which the line segments are equally spaced: such a representative will be called a *unit grid*. The two-cells of a unit grid are squares.

Suppose a rectangular dissection (containing exactly p rectangles, say) is superimposed on the (l, m) unit grid; then it is said to be (l, m) and *standard* if each line of the grid contains at least one edge of a constituent rectangle, and every edge of a rectangle lies on some grid line. The number of rectangular elements, p, is defined here to be the *weight* of the dissection, although elsewhere the terms 'content' and 'order' have also been used for this purpose. If it is assumed, without loss of generality, that $l \leqslant m$, then, for a given p, it is known (Bloch, 1976) that the set of unit-grid dimensions (l, m) to which there correspond (l, m) standard rectangular dissections is

$$\left\{(l, m):\ 1 \leqslant l \leqslant \left\lceil \frac{p}{2} \right\rceil,\ \max\left(\left\lceil \frac{p}{l} \right\rceil, l\right) \leqslant m \leqslant p+1-l\right\},$$

where for any real number s, $\lceil s \rceil$ denotes the least integer greater than or equal to s.

If $l = m$, the symmetry group that leaves the unit grid invariant is clearly D_4, the dihedral group of order eight; whereas, if $l < m$, K_4, the Klein group of order four, is clearly the group that leaves the unit grid invariant. The symmetry of rectangular dissections is taken into account when enumerating them: that is, an equivalence relation is defined under symmetry.