Finite Element Method for Engineers

From Theory to Practice

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Dedication

This book is dedicated to my dear parents Sri. C.R. Velayudhan and Smy. M.K. Devaky their tireless efforts and sacrifices throughout their lives to fulfill their dreams to inspire all the ten children to achieve the highest level of education possible

C.V. Girija Vallabhan

Preface

While there are more than a dozen text books on finite element methods, one might ask why add one more. My response is that I started research on finite element method in early sixties, when there was no text book on finite element method. Since then, during my several years of teaching and research on finite element methods, I developed my own class notes with a beginning student in mind. I accepted a challenge to continuously push myself to present finite element method in a clear and simple format. With that challenge in mind, I updated my class notes in the form of a text book, that would help even students who aspire to learn finite element methods on their own. I synthesized the years of feedback from colleagues and more importantly, from students, to present a beginner's guide to finite element method. This is the purpose of the book.

Most students in civil, mechanical or aeronautical engineering who studied fundamentals of solid mechanics are familiar with the minimum potential energy theorem which is essentially another form of the principle of virtual work or the Castigliano's theorem used in solving structural mechanics problems. It is interesting to note that in the early days of the development of finite element methods, this energy principle was adopted.

Finite element methods in solid mechanics can be introduced in two different approaches: one is by the use of an energy principle and the other is to use a variational technique called the "weak form of the Galerkin principle." Some texts employ the Galerkin principle which is more general and is necessary for solving fluid mechanics problems using finite element method. Since solid mechanics is emphasized here, the main thrust of the methodology adopted is the energy principle. The weak form of the Galerkin principle is discussed at the end of the book to provide a brief review of this principle.

I am extremely thankful to my dear friend and former graduate student, Dr. Mehmet Asik, Professor of Engineering Sciences, Middle East Technical University, Ankara, Turkey, who agreed to be a co-author for the book and helped me to complete the manuscript.

Above all, I am deeply indebted to my brother Dr. Shanmukhan Chiyyarath who pains takingly read the entire manuscript and made valuable suggestions and corrections.

C.V. Girija Vallabhan

Abstract

Organization of the Book

The finite element method can be considered as the application of the classical Ritz method, which is based on the minimum potential energy theorem. The first chapter of the book explains the application of the Ritz method to solve simple (Euler) beam problems in solid mechanics. It is shown that convergence to exact solutions is improved as the number of degrees of freedom in the problem is increased. It is further shown that the Ritz principle can lead to the well known Galerkin principle as well.

Chapter two describes the essential equations in solid mechanics. Also the student is introduced to two-dimensional plane elasticity, axisymmetric and three-dimensional problems in elasticity. An emphasis is made to describe what are known as $\mathbf{C}_0$ and $\mathbf{C}_1$ continuities in functions since there are two different types of finite elements commonly used; one type for analysis of solid continuua and the other for beams, plates and shells.

Chapter three deals with one dimensional simple bar elements (of $\mathbf{C}_0$ type) where all the simple fundamental techniques in finite element method are introduced. The main four steps in the derivation of the stiffness matrix of an element are introduced and emphasized as the same steps can be used for obtaining stiffness matrices of all finite elements. The principle and technique used to assemble elements to form the global stiffness matrix equation, applying boundary conditions by modifying the equations and formation and solution of these equations in a banded form are also presented here. Obtaining equivalent nodal forces using energy principles is also presented here. Higher order bar elements are also introduced. Here the bar elements are extended to analyze two-and three-dimensional trusses. This chapter provides a complete perception of the overall finite element methodology for all other types of finite elements.

Chapter four is dedicated to plane stress/plane strain problems (using $\mathbf{C}_0$ type elements). Here the student gets a first hand feeling for solving a 2-D plane elasticity problems using triangular elements. Higher order triangular elements are introduced in this chapter. Chapter five is for rectangular and quadrilateral finite elements used to solve 2-D plane elasticity problems. Numerical integration schemes using Gaussian quadrature which stimulated the finite element menthodology into developing newer and more complicated finite elements are introduced. Chapter six and seven are devoted for finite element analysis of axisymmetric and 3-D problems in elasticity.

In Chapter eight, beam elements which are of the type with $\mathbf{C}_1$ continuity are discussed. First the familiar Euler beam elements are introduced. Here it is shown that be combining simple bar and beam elements, two-and three-dimensional rigid frames and arches can be analyzed. Further, for the benefit of the beginners in solid mechanics, the Timoshenko beam theory is introduced so that a new beam element can be developed that includes the shear deformation in the beam. Even though this new beam element is not used much for the analysis of beams or frames, the concept used here provides the basis of the plate element that includes the bending and shear deformations in plates. Chapter nine starts with the development of the Classical Kirchhoff plate elements and then later is used to introduce the Mindlin plate elements that are very popular to create quadrilateral elements. Shell elements are introduced in this chapter.

Chapter ten is used to introduce the application of finite elements to solve some two-dimensional steady state flow problems. Towards the end of the chapter the weak form of Galerkin method is presented for those interested in that methodology. Some similarities of the finite method with the classical finite difference method is briefly introduced here.

An Appendix is provided to shown different types of equation solvers that are used for solving finite element equations.

Another important aspect of the book is its emphasis on sign conventions. Since the finite element method extends to many complex three-dimensional problems, there is no doubt that a consistent sign convention is essential. The distinction between the sign convention for bending moments and applied moments is emphasized.

Features of the Book

One of the many features of this book is that it is specifically tailored for self-learning of the finite element method from fundamentals. Basic principles in assembling elements, applying boundary conditions etc. are explained in detail. In addition, it is designed for a one semester course on finite element method. I regularly taught the entire contents of this book in one full semester. This book is not intended to be a references book on finite element method; only a minimum number of references is provided.

In order to understand and feel the thrill of the finite element methodology completely, one has to know how the theory is carried onto the computer through coding. Due to this reason, I am not in favor of introducing commercial finite element codes to beginners. To make students understand the transfer of theory to develop computer code, several simple subroutines are presented that helps them to learn the different computer schemes. Subroutines to compute element stiffness matrix with numerical integration if necessary, to assemble element stiffness matrices to form global equations in a half-banded form, to modify the global equations to satisfy the boundary conditions and to solve for the banded equations are presented. Additionally, the book encourages the student to have a hands-on experience on finite element programming using any one computer language of his/her choice. Fortran language is used in this book. A simple 2-D plane elasticity program is given at the end of this book. By using the methodology that is presented here, one can write a finite element program in any other preferred computer language. Writing one full simple

finite element program is emphasized because the student gets a very clear picture of the organization of all finite element codes that he/she may use in future. Many of my graduate students wrote their own new finite element codes with their own choice of computer language using the information provided here. This experience is extremely valuable to students who comprehend the basic philosophy used in commercial codes. Experience in coding a finite element program helps those who desire to continue research in the area of finite element methods.

C.V. Girija Vallabhan
Mehmet Zülfü Aşık

A Special Acknowledgement

I consider a great honor and privilege to work as the co-author with my former advisor and friend Dr. C. V. Girija Vallabhan who was a distinguished professor with immense knowledge and experience on finite element method. Even though there are many text books on the subject, I agreed to be a coauthor because of the organization and contents of the book which immensely appealed to me. This book is different from others: the methodology followed in the book is logical, descriptive and clarifies the fundamentals of the topic, relates theory, concepts or ideas towards an easy learning of the finite element procedure; it skillfully integrates the knowledge on mechanics, mathematics and numerical computation in engineering. It is a great chance for me to make a contribution for the completion of such a promising book. Again I would like to acknowledge my sincere thanks for allowing me to be a coauthor.

Mehmet Zülfü Aşık

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